PRACTICAL PHYSICS

A LABORATORY MANUAL FOR
COLLEGES AND TECHNICAL SCHOOLS

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COLLEGES AND TECHNICAL SCHOOLS

BY

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VOLUME III

PHOTOMETRY.
EXPERIMENTS IN LIGHT AND SOUND

PREFACE.

The authors believe that physical laboratory work should accompany lecture and recitation work in physics from the beginning of the study of this science in the college or technical school, and they believe that a laboratory manual should set forth a series of definite exercises. A general discussion of how to measure a thing is, in their opinion, not a satisfactory basis for the laboratory work of a student. This laboratory manual has been prepared with these ideas in mind.

A group of students who are beginning a laboratory course on the basis of this manual should be required to study the introductory chapter to Volume I, and to solve the problems illustrating the calculation of probable error.

The system which has been used with great satisfaction by the authors in the assignment of laboratory exercises is described on page 2 of Volume I. The authors believe that one of the most important aspects of laboratory work in physics is that it gives to the student a series of more or less distinctly theoretical problems based upon experimental data obtained by themselves, and in accordance with this idea, the authors believe that a student should be required to work up his laboratory reports outside of the laboratory as specified on page 3 of Volume I.

This manual is issued in three small volumes. The first volume is devoted to Precise Measurements and to Experiments in Mechanics and Heat; the second volume is devoted to Elementary and Advanced Measurements in Electricity and Magnetism; and the third volume is devoted to Photometry and to Experiments in Light and Sound.

The authors wish to acknowledge their obligations to Dr. Howard L. Bronson, of McGill University, for suggestions concerning Experiment 107 on Radio-activity, and the authors'

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thanks are due to Professor Wilbur M. Stein for the use of three cuts from his book on Photometric Measurements, and to Leeds & Northrup for the use of twelve cuts illustrating some of their electrical measuring instruments.

<div align="right">THE AUTHORS.</div>

SOUTH BETHLEHEM, PA.,
December 22, 1907.

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PART VI.

PHOTOMETRY.

LIST OF EXPERIMENTS.

INTRODUCTION TO PHOTOMETRIC MEASUREMENTS.*

Radiant heat. Light. — The radiation from a hot body may be resolved into simple parts each of which is a train of ether waves of definite wave-length. All of these simple parts of the total radiation have one common property, namely, they generate heat in a body which absorbs them. Therefore every portion of the radiation from a hot body is properly called *radiant heat.* The intensity of a beam of radiant heat is measured by the heat it delivers per second to an absorbing body. Thus the radiation emitted by a standard candle represents a flow of about 450 ergs per second across one square centimeter of area at a distance of one meter from the candle.

Radiant heat of which the wave-length lies between 39 and 75 millionths of a centimeter affects the optic nerves and gives rise to sensations of light. Therefore radiant heat of which the wave-length lies between these limits is called *light.* These limits, which are called the limits of the visible spectrum, are not sharply defined, but they vary considerably with different persons and with the degree of fatigue of the optic nerves.

The physical intensity of a beam of light is measured by its perfectly definite thermal effect, that is by the heat energy it delivers per second to an absorbing body. Thus those parts of the radiation of a standard candle which lie within the visible spectrum represent a flow of about 9.3 ergs per second across an area of one square centimeter at a distance of one meter from the candle. Comparing this with the flow of energy which is represented by the total radiation from a standard candle, namely, 450 ergs per second across an area of one square centimeter at a distance of one meter from the candle, it follows that only about 2 per cent. of the energy radiated by a standard candle lies within

* This introduction is adapted from the chapter on photometry of Franklin and Esty's *Elements of Electrical Engineering*, volume I.

the limits of the visible spectrum, that is, only about 2 per cent. of the radiation from a standard candle is light.

The luminous intensity of a beam of light is presumably measured by the intensity of the light sensation it can produce, but the intensity of the light sensation produced by a given beam of light is extremely indefinite. A given beam of light entering the eye may produce a strong or a weak sensation depending upon manifold individual peculiarities of the person and upon the degree of fatigue of the retina, and the vividness of the sensation depends upon the extent to which it is enhanced by attention. Our sensations are really not quantitative in the physical meaning of that term ; in fact they enable us merely to distinguish objects, to judge whether things are alike or unlike, and the certainty and precision with which we can do this is exemplified in every outward aspect of our daily life. *The ratio of the luminous intensities of two beams of light is measured by using a device to alter* **in a known ratio** *the physical intensity of one beam until it gives, as nearly as one can judge, a degree of illumination on a screen which is equal to (like) the illumination produced by the other beam.*

Photometry. — The measurement of the light emitted by a lamp is called *photometry.* This measurement is always made by comparing a beam of light from the given lamp with a beam of light from a standard lamp, as explained in the previous paragraph, and the physical device there referred to is called a *photometer.* The comparison of the total light in a beam from a given lamp with the total light in a beam from a standard lamp is called *simple photometry ;* whereas the comparison, wave-length by wavelength, throughout the spectrum is called *spectro-photometry.* A fundamental difficulty in simple photometry is that different lamps usually show differences of color, and these differences of color do not disappear when the attempt is made to adjust a photometer to equality of brightness.

Standard lamps. — The *British standard candle* is a sperm candle made according to exact specifications.* When this

* See *American Gas Light Journal,* Vol. 60, page 41, 1894.

candle burns 120 grains of sperm per hour, it is a standard candle, and the actual candle power during a given test is defined as $a/120$, where a is the number of grains of sperm actually burned per hour. The British standard candle is now obsolete.
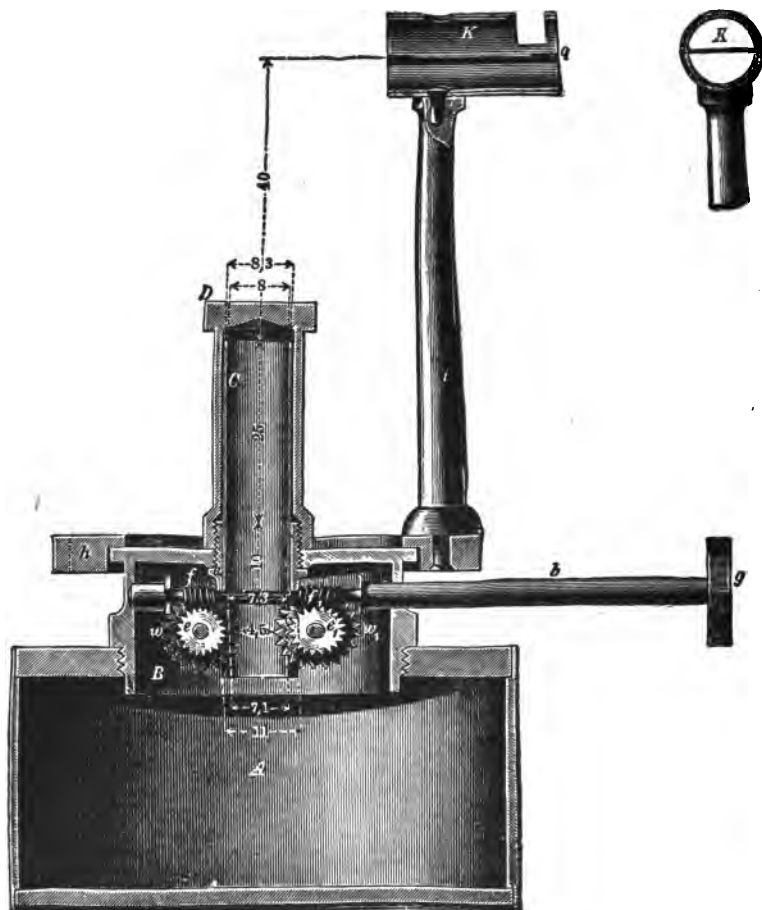
Fig. 147.

The Hefner lamp, so-called from its inventor, is a lamp which burns pure amyl acetate, the wick and its containing tube are of prescribed dimensions, and the wick is turned up to give a flame

of prescribed height. Figure 147 is a sectional view of the Reichs-anstalt form of the Hefner lamp, showing the standard dimensions in millimeters. The height of the flame is indicated by a sighting arrangement K. Most of the lamps are also provided with a Krüss optical flame-gauge, which consists of a simple lens and a ground glass screen upon which the image of the top of the flame is projected. The Krüss gauge is shown in Fig. 148.
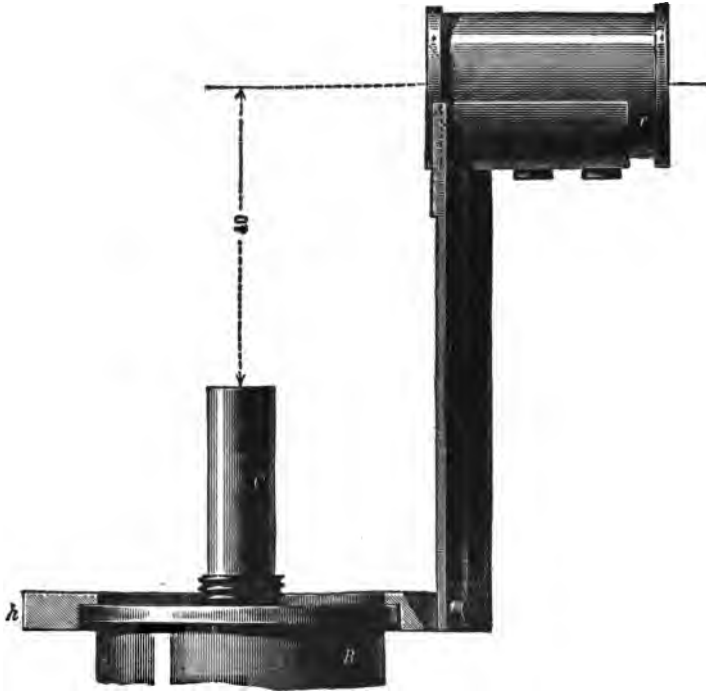
Fig. 148.

Figure 149 shows a test gauge intended to be placed over the wick tube, when there should be the slightest perceptible clearance above the top of the wick-tube as seen through the slit s, and the extreme top of the gauge should be in line with the axis of the flame-gauge K, Fig. 147.

A full discussion of the Hefner lamp may be found in *Photo-*

metric Measurements by Wilbur M. Stine, The Macmillan Company, 1900. In particular see discussion of influence of atmospheric moisture, influence of carbon dioxide, influence of atmospheric pressure, and influence of atmospheric temperature, on pages 153 to 157.

In using the amyl acetate lamp, trim the top of the wick square and smooth, unscrew the top of the lamp, pour in the amyl acetate, and allow the lamp to burn freely for at least 10 minutes before using it in making measurements. On the top plate near the wick tube are a few small vent holes which must be inspected and kept open. The temperature of the photometer room should be between 15° and 20° C. The lamp must not be used in a small or poorly ventilated room on account of the accumulation of moisture and carbon dioxide.

When the measurements are finished, the lamp must be emptied and cleaned, on account of the corrosive action of the amyl acetate on the metal parts of the lamp and the lamp and wick tube must be rinsed with alcohol and allowed to dry. The wick itself should be washed in two or three changes of alcohol and allowed to dry.

Fig. 149.

Light units. — The intensity of a horizontal beam of light from a Hefner lamp is called a *hefner unit* or a *hefner*. If a lamp were to give one hefner unit of light intensity in every direction, the amount of light, or the so-called flux of light emitted by the lamp, would be what is called one *spherical-hefner*.

The great reliability of the Hefner lamp as compared with the standard sperm candle has led to the definition of the candle in terms of the hefner unit. The *candle* or the *candle unit* is a beam of light of which the intensity is 1.136 hefner units and the *spherical-candle* is equal to 1.136 spherical-hefners. The standard sperm candle is no longer used in photometric work.

Conical intensity and sectional intensity of light. — The ex-

pression, *intensity of a beam of light*, which is used in the above definitions of the hefner and the candle, refers to the amount of light in a unit-sized cone of rays. This *conical intensity*, which it may be called for brevity, is expressed in hefners or candles; it is independent of distance, since the light in a given cone of rays remains in that cone; and it depends only upon the area and brightness of the luminous surface of the lamp.

The intensity of a beam of light may also refer to the amount of light per unit sectional area of the beam. This *sectional intensity* of a beam of light, which it may be called for brevity, decreases as the square of the distance from the lamp increases, and it is expressed in terms of a unit called the *lux*, which is the sectional intensity of a horizontal beam from a Hefner lamp at a distance of one meter from the lamp. The sectional intensity I of a given beam in luxes at a distance of d meters from a lamp is given by the equation :

$$I = \frac{h}{d^2} \qquad\qquad (\text{i})$$

in which h is the conical intensity of the beam in hefner units.

Intensity of illumination. — The amount of light per unit sectional area of a beam, that is, the sectional intensity of the beam, measures the intensity of illumination of a surface upon which the beam falls perpendicularly. Therefore the intensity of illumination of a surface may be expressed in luxes. Thus the intensity of illumination required for easy reading is the intensity of illumination at a distance of one foot from a standard candle, which intensity is sometimes called the " candle-foot," and it is equal to $1.136/(0.305)^2 = 12.21$ luxes according to equation (i).

A complete statement of the various photometric units depends upon a clear understanding of what is called solid angle. Consider a cone, and a sphere with its center at the apex of the cone. The solid or spherical angle of the cone is measured by the ratio of the area of the portion of the spherical surface within the cone to the square of the radius of the sphere. Thus the unit of solid angle is subtended by one square centimeter of the surface of a sphere of one centimeter radius, and a complete surface of a sphere represents 4π units of solid angle.

Consider a lamp placed at the center of a sphere of unit radius so that one unit of area of this sphere may represent one unit of solid angle or one *unit-cone*. Imagine

the lamp to give one hefner of conical intensity of light in every direction. Then the amount of light (light flux) passing out in one unit-cone (through unit area of the sphere) is called one *lumen* of light flux.

The given lamp emits one spherical-hefner (0.88 of a spherical-candle) of light flux, because the conical intensity is assumed to be the same in every direction; but the whole spherical surface represents 4π units of solid angle or 4π unit cones. Therefore the lamp emits 4π lumens of light flux. That is to say, a spherical-hefner of light flux is equal to 4π lumens.

The lux, which is defined above as the intensity of illumination at a distance (horizontally) of one meter from a Hefner lamp, represents, of course, a certain amount of light flux falling upon each square centimeter of the illuminated surface. Now the area of a sphere of one meter radius is $40,000\pi$ square centimeters and if the lamp gave out light equally in all directions, one spherical-hefner or 4π lumens would pass out from it. Therefore one lux represents one ten-thousandth of a lumen per square centimeter or $\frac{1}{40000\pi}$ of a spherical-hefner per square centimeter.

As an illustration of the significance of the terms hefner, lumen and lux, consider a beam of light emitted by a glow-lamp. Let the conical intensity of this beam be 18.2 hefners (16 candles). Let the solid angle of this beam be 0.01 of a unit, that is to say, the solid angle subtended by 0.01 of a square centimeter of the surface of a sphere of which the radius is one centimeter. Then the number of lumens of light flux in the beam is 18.2 hefners multiplied by 0.01 unit of solid angle, which is equal to 0.182 lumen. The sectional intensity of this beam at a distance of one meter from the lamp is 18.2 luxes.

The so-called search-light presents some very interesting features in regard to conical intensity. The numerical example given below will serve as an illustration. To understand this example one must remember that light which emanates from a *point* in the focal plane of a lens or mirror is transformed by the lens or mirror into a beam of rays which are all parallel (ignoring errors of spherical and chromatic aberration and of astigmatism) to a line drawn from the point to the center of the lens or to the center of curvature of the mirror. Therefore the light which emanates from a *small luminous surface* in the focal plane is transformed into a series of parallel beams all the rays of which lie (at a great distance from the search-light) within the cone formed by drawing lines from every point of the small luminous surface to the center of the lens or to the center of curvature of the mirror, and the solid angle of this cone is equal to the area of the small luminous surface divided by the square of the focal length of the lens (or mirror).

The powerful arc lamp of a certain search-light emits light of 10,000 candles conical intensity towards every part of a lens (or mirror) which subtends one unit of solid angle as seen from the arc which is at its focus. The luminous surface of the lamp is one quarter of a square centimeter and the focal length of the lens (or mirror) is 30 centimeters. Therefore the solid angle of the cone which contains the search-light beam is ¼ divided by 30^2 or $\frac{1}{3600}$ of a unit, and, if we assume that no light is lost in the lens (or on the mirror) by absorption, the conical intensity of the search-light beam must be 3,600 times as great as the conical intensity of the light direct from the lamp, or 36,000,000 candles. One who considers this example carefully will be impressed with the important fact that the candle is not a unit of quantity of light. The candle and the hefner are units of conical intensity.

The Bunsen photometer is a device for measuring the conical intensity of a beam of light from a given lamp in hefners or in candles. It is the photometer that is almost universally used in simple photometric measurements. The given lamp and the standard lamp are placed at the ends of a horizontal bar and a screen of thin unsized paper is moved along the bar until the two sides of the screen are equally illuminated by the two lamps. The intensities of illumination (sectional intensities of the beams of light) due to the respective lamps are h/d^2 and h'/d'^2, according to equation (i), and since these are equal we have :

$$\frac{h}{h'} = \frac{d^2}{d'^2} \qquad\qquad \text{(ii)}$$

in which h and h' are the conical intensities in hefners or in candles of the light sent towards the screen by the respective lamps, and d and d' are the distances of the lamps from the screen.

An irregular grease spot on the thin paper screen enables one to judge better when the illumination is the same on the two sides. This spot should be made with clean paraffin and the excess of paraffin should be drawn out of the screen by placing it between folds of absorbent paper and applying a hot flat-iron.

The bar of the Bunsen photometer is generally divided to read the value of the ratio of the sectional intensities of the beams of light from the two ends (h/h'), and the product of this reading and the intensity of the beam from the standard lamp gives the intensity of the beam from the other lamp.

In judging the equality of the illumination on the two sides of the Bunsen photometer screen, it is important to use only one eye. In using two eyes one unconsciously looks at one side of the screen with one eye and at the other side of the screen with the other eye, and the difference between the two eyes leads to a constant error of setting.

In the use of the Bunsen photometer a carbon-filament electric lamp is generally used as a working standard. This lamp is

previously standardized for a particular voltage and in a particular direction, by comparing it with a Hefner lamp: and when used it is operated at this particular voltage. Carefully standardized carbon-filament lamps are offered for sale by the United States Bureau of Standards, Washington, D. C.

Distribution of light around a lamp. — In the definition of the spherical-hefner the idea of uniformity of distribution of light around the lamp was introduced for the sake of simplicity.   In fact, however, no lamp gives complete uniformity of distribution,
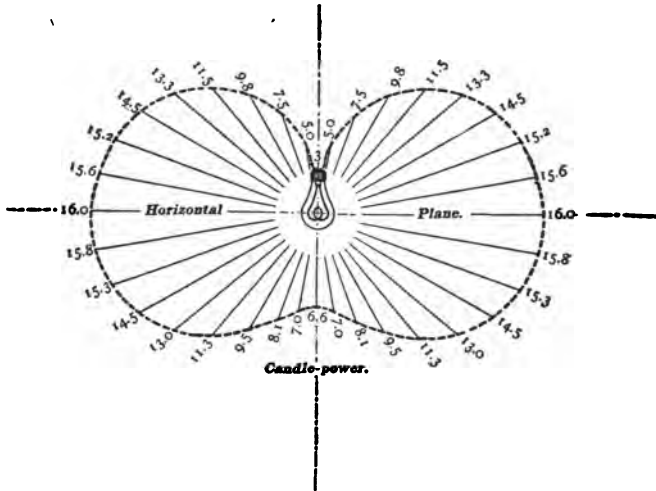
Fig. 150.

but the conical intensity in hefners or candles is always greater in certain directions and less in other directions.   Thus Fig. 150 * shows the distribution of light about a typical " 16-candle-power " carbon-filament lamp without a shade, and Fig. 151 shows the distribution of light about the same lamp when it is placed in an aluminum cone reflector.   In these figures the conical intensity of the light in each direction in candles is represented to scale by the length of the corresponding radius vector of the dotted curve.

* Taken from a paper by J. R. Cravath and V. R. Lansingh on Reflectors, Shades and Globes, *Electrical World and Engineer*, Vol. 46, pp. 907, 947, 991, 1033 and 1074, November 25 to December 23, 1905.

The distribution of light about a lamp, which, like a carbon-filament electric lamp, can be held in any position, may be determined by mounting the lamp in a universal holder at one end of the photometer bar, turning it step by step into various positions, and taking the photometer reading for each position.

In some cases a lamp is symmetrical with respect to an axis, so that a complete knowledge of the distribution of light about the lamp may be obtained by determining the intensities of the light in different directions in a single plane which contains the axis of symmetry.

Fig. 151.

In many cases a lamp is approximately symmetrical with respect to an axis so that the slight variations of the intensity of the light around the axis of approximate symmetry are of no importance. In such a case the lack of symmetry may be averaged out, as it were, by rotating the lamp at a speed of three or four revolutions per second about its axis of approximate symmetry while the photometric readings are being taken. The data for Figs. 150 and 151 were obtained in this way.
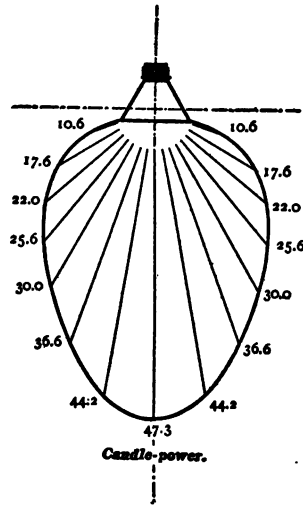
In the case of a lamp which must be held in a fixed position, one or more mirrors are used to reflect the different beams from the lamp along the photometer bar. Thus Fig. 152 shows three mirrors AA, BB and CC arranged to reflect the light from a fixed lamp, L, along a photometer bar. The three mirrors are supported in a rigid frame which may be rotated about the line DE as an axis. The figure shows the mirrors in the position to reflect the downward beam from the lamp to the photometer screen.

The mirrors AA, BB and CC, Fig. 152, must be large enough so that with the eye placed at the photometer screen one

can see the entire luminous surface of the lamp including the globe or shade ; and the distance of the lamp from the screen must be taken as the sum of the distances f, g, h and i, Fig. 152.

The mirrors reflect a certain fractional part, only, of the light from the lamp, and, therefore, the photometer reading must be multiplied by a correction factor.   This correction factor is found
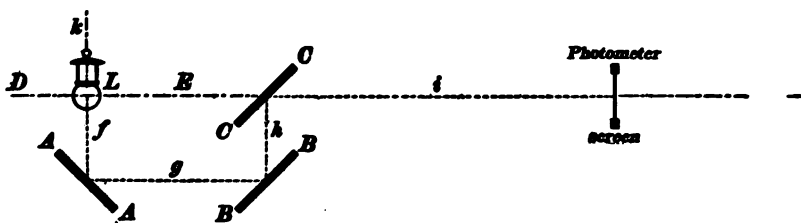
Fig. 152.

by observing the photometer readings of the horizontal beam from the lamp with and without the mirrors, making due allowance for the effective distance from lamp to screen in each case.

If it is feasible, the lamp should be rotated steadily about the vertical axis kf, in Fig. 152, while the photometer readings are being taken.

Measurement of total light flux from a lamp. — If a lamp were to emit light of the same conical intensity in all directions, then the conical intensity of the light in hefners (or candles) would be numerically equal to the total light flux from the lamp in spherical-hefners (or spherical-candles), and a single measurement of such a lamp by means o. a Bunsen photometer would give not only the conical intensity of the light in hefners (or candles) but also the total light flux in spherical-hefners (or spherical-candles).*

In general, however, light is emitted by a lamp unequally in different directions and it is necessary to distinguish between con-.

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* There is a widespread tendency among engineers to confuse the unit of conical intensity, the candle, with the unit of light flux, the spherical-candle.   This is due to the fact that, in the absence of exact data concerning the distribution of light around a given lamp, the irregularities of distribution are ignored, and the intensity of a horizontal beam in candle-power is used as the easiest and simplest approximate measure of the total light given off by the lamp, as if the light were of the same conical intensity in all directions.

ical intensity and total light flux. The total light flux in spherical-hefners emitted by a lamp is determined by measuring the conical intensity in hefners in every direction and taking the average, which gives the light flux in spherical-hefners. *If this average is to be calculated in the ordinary way by adding and dividing, the directions in which the separate readings are taken must be distributed uniformly over the surface of a sphere with its center at the lamp.* This sphere is called the reference sphere for brevity. If the readings are not so distributed, each reading must be multiplied by the spherical area which may be properly assigned to it, and the sum of such products must be divided by the total area of the reference sphere to give the correct average.

When a lamp can be rotated at a speed of three or four revolutions per second about its axis of approximate symmetry, the total light flux may be determined accurately by taking readings of the conical intensity in different directions in one plane only, namely, a plane which includes the axis of rotation. Thus the lamp L, Fig. 153, is rotated about the vertical axis PQ, and the conical intensities R, R', R'', R''', etc., at equal angular distances ϕ are measured. On account of the rotation of the lamp each setting of the photometer gives the average conical intensity along a parallel of latitude, as it were. Each reading R, R', R'', etc., represents, therefore, the conical intensity over a zone of the reference sphere; so that the readings must be

Fig. 153.

multiplied by the areas of the respective zones, and the sum of these products must be divided by the total area of the reference sphere to give the average conical intensity in all directions, which is equal of course to the total light flux in spherical-units, hefners
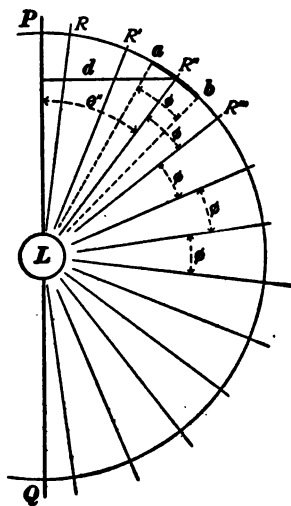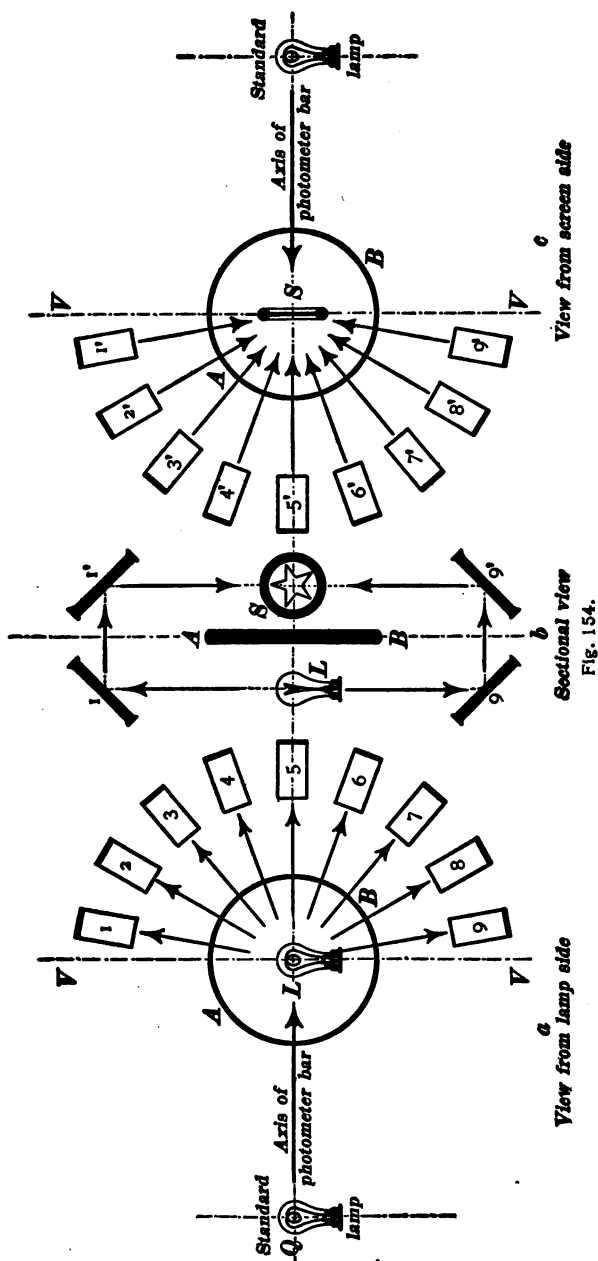
or candles as the case may be. Thus the reading R'', Fig. 153, refers to the spherical zone ab of which the area is sensibly equal to the length of the arc ab ($= r\phi$) multiplied by the mean circumference ($= 2\pi d = 2\pi r \sin \theta''$) of the zone. The radius r of the reference sphere cancels out when one divides the above-mentioned sum by the total area of the reference sphere ($= 4\pi r^2$), as above explained.

Example. — The conical intensities are given in Fig. 150 for every 10°. The area of each of the two polar zones corresponding to conical intensities 3.0 and 6.6 is, $\pi/4 \times (\frac{10}{360}$ of $2\pi r)^2$: the area of each of the zones corresponding to conical intensities 5.0 and 7.0 is, $2\pi r \sin 10° \times (\frac{10}{360}$ of $2\pi r)$; the area of each of the zones corresponding to conical intensities 7.5 and 8.1 is, $2\pi r \sin 20° \times (\frac{10}{360}$ of $2\pi r)$; and so on. A calculation of the mean conical intensity in all directions from the data given in Fig. 150 gives 13.33 spherical candles as the light flux emitted by the lamp.

The Matthews integrating photometer. — Two ingenious arrangements of mirrors have been devised by Professor C. P. Matthews by means of which the light flux from a lamp in spherical-candles can be determined by a single setting of a photometer.* A photometer provided with such an arrangement of mirrors is called an *integrating photometer*. The essential features of one of Professor Matthews' arrangements are shown in Fig. 154. The lamp L to be tested is rotated about the vertical axis VV and a number of equidistant beams from L, all lying in the plane QLV and each representing a zone of the reference sphere, are reflected to the photometer screen S, as shown. This photometer screen is stationary and the photometer setting is made by moving the standard lamp nearer to or farther from the screen.

If the beams of light from all of the mirrors struck the photometer screen at right angles, the reading of the photometer

* The Integrating Photometer, C. P. Matthews, *Transactions of American Institute of Electrical Engineers*, Vol. XVIII., pp. 677–697, 1901 ; Vol. XX., pp. 59–70, 1902.

Standard lamp

Axis of photometer bar

View from screen side

c

Sectional view

Fig. 154.

b

Standard lamp

Axis of photometer bar

View from lamp side

a

would be the sum of the conical intensities of all the beams, that is, the photometer reading would be *proportional* to the *simple average* of the conical intensities of all of the beams.

The spherical-candle power of a lamp is found, however, not by taking the simple average of the conical intensities in various directions, but by taking what is called a *weighted average* as explained on page 13.

If the illumination of the screen by each beam alone were z'/z as great as it would be if the beam struck the screen at right angles, z being the area of the equatorial zone of the reference sphere which is associated with the beam that is at right angles to VV, and z' being the area of that zone of the reference sphere which is associated with any given beam, then the reading of the photometer, Fig. 154, would be the sum $\Sigma(z'I/z)$ that is, the photometer reading would be proportional to the true spherical-candle power of the lamp. In fact, the reading of the photometer would be n times the spherical-candle power $(=\Sigma z'I/4\pi r^2)$, where n is equal to the ratio of the area of the entire reference sphere $4\pi r^2$ to the area of the equatorial zone z. The incomplete reflection of light by the mirrors is considered later.

If the photometer screen were entirely free from gloss an oblique beam of light would produce a degree of illumination inversely proportional to the area of screen over which unit sectional area of the oblique beam is spread, that is, a degree of illumination proportional to the sine of the angle between the beam and the plane of the screen. This is precisely the reduction of illumination specified above in terms of the ratio z'/z. It is impossible however to make a photometer screen without gloss, that is, a screen that does not show regular reflection to some extent, and the more oblique the incident light the larger the proportion of the light which is reflected regularly and the less the proportion that is reflected diffusely by the screen. Therefore the mirrors near the axis VV, in Fig. 154, must be somewhat nearer to L and S so as to lessen the optical distance from L to S, and thus intensify slightly the beams that strike the screen obliquely.

The Matthews integrating photometer is adjusted as follows so as to eliminate errors due to gloss of screen and errors due to absorption of light by the mirrors. Put an auxiliary standard lamp in place of L, cover all of the mirrors except the pair that receives the horizontal beam from the auxiliary standard, that is, mirrors 5 and 5′ in Fig. 154, and take the reading k* of the photometer. Then cover all of the mirrors except any given pair, turn the auxiliary standard lamp so that its standard face or aspect is towards the uncovered pair, set the photometer arbitrarily at reading kz'/z, and adjust the uncovered pair of mirrors inwards or outwards until the screen shows equal illumination on its two sides. Proceed in like manner with each pair of mirrors until the adjustment is complete.

When the photometer has been thus properly adjusted its reading must be multiplied by the factor $z/(4\pi kr^2)$ to give the true spherical-candle power of a lamp L which is being measured, z being the area of the zone of the reference sphere which is associated with the horizontal beam, and $4\pi r^2$ being the total area of the reference sphere.

The flicker photometer is a device for eliminating to some extent the error in setting a photometer due to differences of color of the lamps which are to be compared. The following is the principle upon which the elimination of color error is based : When one looks at a thing such as a photometer screen, one has a sensation of *brightness* and a sensation of *color*. Both of these sensations persist for an appreciable interval of time after light ceases to enter the eye, but sensations of color persist longer than sensations of brightness. If the two sides of the photometer screen are brought into the same field of view in rapid succession or with high frequency of interchange, the color sensations produced by the two sides of the screen and also the brightness sensations produced by the two sides of the screen will be completely blended. A much lower frequency of interval will suf-

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* The quantity $1/k$ is the factor by which any reading of the photometer must be multiplied to correct for loss of light due to incomplete reflection by the mirrors.

fice to blend the color sensation but leave a flickering sensation of brightness unless the two sides of the screen have the same brightness. Therefore if a fairly low frequency of interchange be used, then the two sides of the photometer screen can be brought to equal brightness by adjusting the photometer until the flicker disappears.

The device employed by Whitman * in his Flicker Photometer is as follows : A white cardboard disk A, Figs. 155 and 156, is
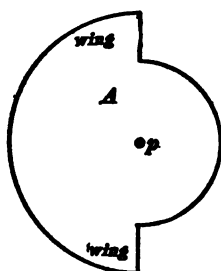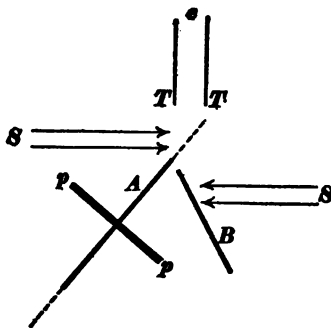
Fig. 155.   Fig. 156.

mounted on the axis pp, and a stationary piece of the same cardboard is placed at B, Fig. 156. When the disk A is rotated, the wings of the disk and the cardboard B are seen in succession by the eye placed at e and shielded by the tube TT. The wing of the disk is illuminated by light from the source S_1, and the cardboard B is illuminated by the source S_2. The disk A is driven at sufficient speed to blend the color sensations, and then the distances of the two lamps S_1 and S_2 are adjusted until the flickering sensation of brightness disappears.

The spectrophotometer is a combination of a spectroscope and a photometer for comparing the intensities of two beams of light, wave-length by wave-length. A simple form of spectrophotometer is shown in Figs. 157a and 157b. Figure 157a shows a standard lamp and a given lamp L at the ends of a photometer

* F. P. Whitman, *Physical Review*, Vol. 3, page 241.

bar. Two total reflecting prisms *PP*, Fig. 157*a*, reflect light from the respective lamps into the slit *SS* of a spectroscope, so that the spectra of the two lights are seen side by side in the spectroscope. Figure 157*b* shows the spectroscope (direct-vision variety) mounted on a car so that it can be easily moved along
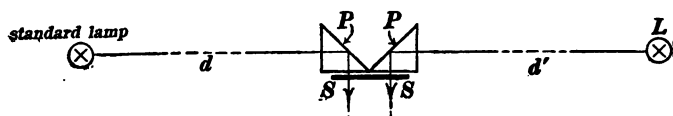
Fig. 157a.

the photometer bar, thus changing the relative intensity of the two spectra at will. Observations are taken with the instrument as follows : The observer's attention is fixed on a certain region of the two spectra say, the extreme red, and the car is moved until the two spectra are of the same intensity in this region, then the ratio of brightness of the two lights for this region of the spectrum is equal to the ratio of the squares of the distances *d* and *d'* in Fig. 157*a*.* This operation is repeated step by step throughout the spectrum. The attention of the observer is directed to a certain region of the two spectra by plac-

Fig. 157b.

ing in the focal plane of the telescope a diaphragm with a narrow slit in it so that only a narrow strip of the two spectra is visible.

The objection to the spectrophotometer above described is that the two luminous fields which are compared (the two parts of a narrow slit in the eye-piece of the telescope of the spectrophotometer) are very small. This difficulty is obviated in the spectro-

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* Each of these distances should be measured as the total length along the path of the rays from the lamp to the slit.

photometer devised by Lummer and Brodhun, the essential features of which are shown in Fig. 158. Two right-angled prisms *XX* are cemented together forming a cube, one prism being cut away slightly so as to leave a central patch *c* in actual contact, whereas the portions *dd.* of the prisms do not come into contact. The light from the standard lamp *L'* passes through a slit *S'*, through the collimating lens *C'*, is totally reflected from the portions *dd* into the prism *P* of the spectroscope, whereas the central portions pass through *c* without any reflection at all.
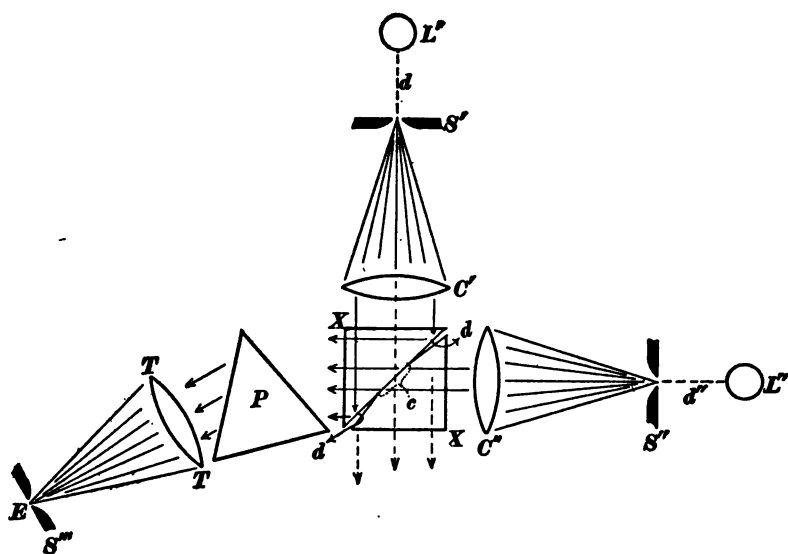
Fig. 158.

Light from the other lamp *L''* passes through a slit *S''*, through the collimating lens *C''*, and through the central portion *c* into the prism *P* of the spectroscope, the edge portions of the light from *L''* being totally reflected by the portions *dd* of the double prism *XX*. After passing through the prism *P*, the light is brought to a focus by the telescope objective *TT*, and a slit *S'''* is placed at the focus of this lens so that light of a single wavelength enters the eye at *E*, and the eye, focussed upon the face *dcd* of the double prism *XX*, sees the central portion of the field

uniformly illuminated by light of one wave-length from L' and the edge portions of the field uniformly illuminated by light of one wave-length from L''. The distances d' and d'' are then adjusted until the field is uniformly bright, when the intensity of the given wave-length from L' is to the intensity of the same wave-length from L'' directly as the squares of the distances d' and d''.

Example of spectrophotometric measurements. — The results of the spectrophotometric comparison of gaslight, limelight, and daylight are shown by the curves in Fig. 159.* The curves refer to beams of gaslight, limelight and daylight, all of which have the same intensity as Fraunhofer's D line (the middle of the yellow region of the spectrum); and the

Fig. 159.

curves show, for example, that the beam of daylight is about six times as bright as the beam of gaslight at Fraunhofer's F line (in the blue region of the spectrum), and only about one fourth as bright as the gaslight at Fraunhofer's B line (in the red region of the spectrum).

EXPERIMENT 109.
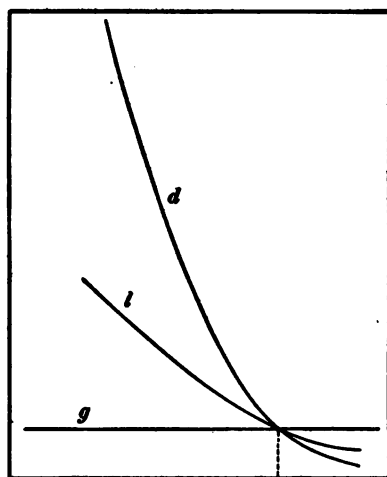
DETERMINATION OF MEAN HORIZONTAL CANDLE POWER.

The object of this experiment is to determine what is called the mean horizontal candle power of a lamp.

Apparatus. — The lamp to be tested is mounted at one end of

* See Nichols and Franklin. A spectrophotometric comparison of sources of artificial illumination, *American Journal of Science*, Vol. 38, pp. 100-114, December, 1889.

the photometer bar and rotated at a speed of from three to four revolutions per second about a vertical axis; a standard lamp is placed at the other end of the photometer bar; and the setting of the photometer then indicates the mean horizontal candle power of the given lamp.

Figure 160 shows a rotating holder for an ordinary glow-lamp. The axis of rotation *A* of the lamp can be adjusted to any desired inclination by turning the plate *D*. This plate contains a number of holes in its edge into any one of which the bolt *B* may be inserted to hold the plate in the desired position, and the face of *D* is divided so that the angle of inclination of the axis of rotation of the lamp can be read off directly.

If the lamp cannot be rotated, its mean horizontal candle power must be determined by turning the lamp step by step about a vertical axis, observing its candle power in each position and taking the average of these results.

It is customary to rate incandescent electric lamps on the basis of their mean horizontal candle power, and it is customary to specify what is called the efficiency of an incandescent lamp by giving the power consumption in watts per mean horizontal candle power; thus, a 3.1 watt per candle lamp would be one consuming 3.1 watts for each unit of mean horizontal candle power. It is important not to confuse horizontal candle power with mean spherical candle power.

Work to be done. — (*a*) *To determine the consumption of kerosene per unit of mean horizontal candle power of an ordinary kerosene lamp and to determine the curve of distribution of candle power in a horizontal plane about the lamp.* Trim and clean the lamp, fill it with kerosene and weigh it carefully. Then place it upon the photometer bar, light it and note the clock reading.

After the lamp has burned for ten or fifteen minutes, turn it step by step about a vertical axis and take repeated readings of the photometer. The lamp should be turned through one eighth or one sixteenth of a revolution between successive readings and forty or more readings should be taken in all.

At the end of the run, note the clock reading, and weigh the lamp again carefully so as to determine the consumption of kerosene.

(*b*) *To determine the watts consumed by an electric incandescent lamp per unit of mean horizontal candle power.*   Place the lamp on the rotating spindle at the end of the photometer bar, arrange an ammeter and a voltmeter for measuring the power delivered to the lamp, and observe the mean horizontal candle power, the axis *A* of rotation of the lamp being vertical, as shown in Fig. 160.

(*c*) *To determine the intensity of illumination required for reading newspaper print.*   Measure the greatest distance from a standard lamp at which newspaper print can be easily read in a dark room with black walls.

Computations and results. — From the data obtained under (*a*) calculate the mean horizontal candle power of the kerosene lamp, and calculate the consumption of kerosene per hour per unit of mean horizontal candle power.   Also plot a curve showing the distribution of light about the kerosene lamp in a horizontal plane.

From the data obtained under (*b*) calculate the watts consumed by the electric incandescent lamp per unit of mean horizontal candle power.

From the data obtained under (*c*) calculate the intensity of illumination in luxes (and also in terms of the " candle-foot ") required for easy reading of newspaper print.

EXPERIMENT 110.

DETERMINATION OF TOTAL LIGHT FLUX. ABSORPTION OF LIGHT BY A SHADE.

The object of this experiment is to determine the total light flux emitted by a lamp, to investigate the effect of a shade in modifying the distribution of this light, and to determine the percentage of light which is lost by absorption of the shade.

Apparatus. — This experiment is to be applied to an electric incandescent lamp mounted on a rotating axis, the axis being arranged so that it can be placed at any angle with respect to the axis of the photometer bar, as shown in Fig. 160, thus avoiding the use of mirrors as represented in Fig. 152.
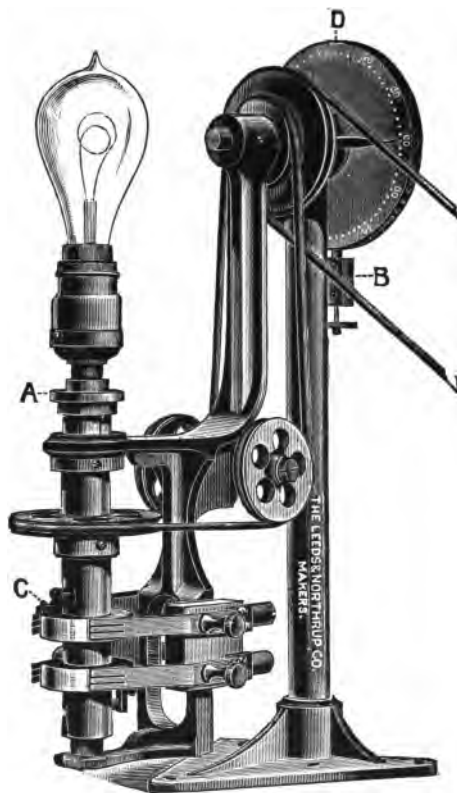

Fig. 160.

Work to be done. — (*a*) Place the bare lamp (without its shade) in position, set it spinning about its axis of approximate symmetry, and read the photometer for a series of observed angles between the axis of revolution of the lamp, and the axis of the photometer bar, as explained on page 13.

(*b*) Place the shade on the lamp and repeat (*a*).

These two sets of observations should be repeated one after the other several times to eliminate errors due to unavoidable fluctuations of the lamps, and in some measure to eliminate other erratic errors.

Computations and results. — Plot curves similar to Figs. 150 and 151, showing the distribution of light about the lamp without a shade and with a shade. Calculate the light flux from the lamp in spherical-candles without a shade and with a shade, and determine the percentage of light lost in the shade.

EXPERIMENT 111.

ADJUSTMENT AND USE OF THE INTEGRATING PHOTOMETER.

The object of this experiment is to afford an exercise in the adjustment and use of the Matthews integrating photometer.

Apparatus. — The type of integrating photometer that is to be used in this experiment is described on page 14 and represented in Fig. 154.

Work to be done. — (*a*) Adjust the photometer as explained on page 17.

(*b*) Measure the spherical-candle power of an incandescent electric lamp as explained on page 17.

EXPERIMENT 112.

THE USE OF THE SPECTROPHOTOMETER.

The object of this experiment is to afford an exercise in the use of the spectrophotometer by determining the curve of absorption of light by a colored liquid.

Work to be done. — Using two exactly similar lamps in the arrangement shown in Figs. 157 or 158, place a vessel of colored liquid (a solution of potassium bichromate, for example) so that the light from one lamp passes through it.

Take settings of the photometer at various points throughout the spectrum as indicated by the reading of the circle which is attached to the spectroscope.

Results. — Plot a curve of which the abscissas are these circle readings and of which the ordinates are the fractional parts of each wave-length of light which are transmitted by the colored solution.

EXPERIMENT 113.

LUMINOSITY CURVE OF THE PRISMATIC SPECTRUM OF LAMPLIGHT.

The object of this experiment is to determine the relative luminosities of the different parts of the spectrum of lamplight.

Theory. — The intensity of the light sensation produced by a beam of light is called the *luminosity* of the light, and it does not by any means bear a constant ratio to the intensity of the beam as measured by its thermal effect. Thus, a beam of yellow light of a given thermal intensity has a much greater luminosity than a beam of red light of the same thermal intensity. For the purposes of the following discussion it is convenient to speak of the luminosity of a given wave-length of light of unit thermal intensity as the *specific luminosity* of that particular wave-length.

The relative luminosities of the different parts of the spectrum of the light from a lamp depend in part upon the specific luminosities of the different wave-lengths, in part upon the relative thermal intensities of the different wave-lengths, and in part upon the manner in which a prism crowds certain portions of the spectrum together and spreads other portions of the spectrum apart. Thus, the blue part of the spectrum of lamplight has a very weak luminosity as compared with the yellow because the actual energy of a given wave-length in the blue is much less than the energy of a given wave-length in the yellow; because the blue region of the spectrum greatly dispersed by a prism as compared with the yellow, and therefore a given wave-length interval is spread over a much greater space in the blue than in the yellow; and because the specific luminosity of yellow light is different from the specific luminosity of blue light.

Apparatus. — The arrangement to be used in this experiment is an ordinary spectroscope and a rotating sectored disk constitut-

ing a flicker photometer.* Two similar lamps L' and L'' are arranged as shown in Fig. 161 ; the light from L' passes through the slit of a spectroscope in the ordinary way. DD is a rotating disk of cardboard similar to Fig. 155 and pp is its axis of rotation. In the focal plane ff of the spectroscope is a narrow slit s which exposes only a narrow portion of the spectrum to view, and the distance d of the lamp L'' from the point W
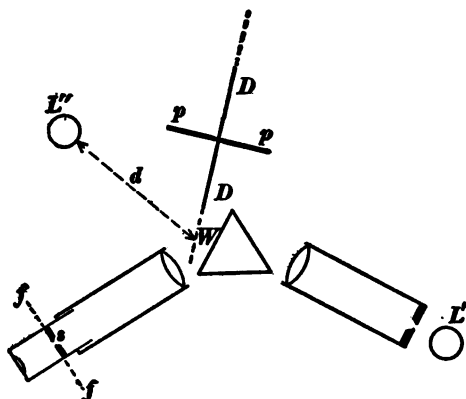

Fig. 161.

where the wing of the disk comes in front of the telescope is adjusted until the flicker of light in the narrow slit s disappears. This adjustment is repeated for each part of the spectrum and the relative luminosities are then proportional to the inverse squares of the distances d.

Work to be done. — Determine as explained above the luminosity curve of the prismatic spectrum of the light from a kerosene lamp and plot the results, using positions in the spectrum as indicated by the circle readings of the spectrometer as abscissas, and luminosities measured by the inverse squares of the distances d as ordinates.

* This arrangement was described by Dr. F. L. Tufts before the American Physical Society in October, 1907.

PART VII.

EXPERIMENTS IN LIGHT AND SOUND.

LIST OF EXPERIMENTS.

EXPERIMENT 114.

RADIUS OF CURVATURE OF A CONCAVE MIRROR BY REFLECTION.

The object of this experiment is to determine the radius of curvature of a concave mirror.

Theory. — A circular mirror MM, Fig. 162, with its center
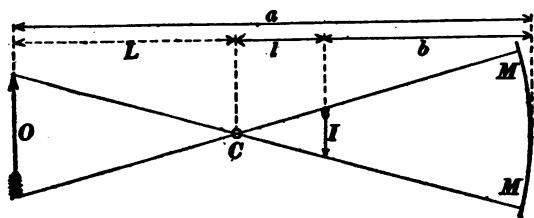
Fig. 162.

of curvature at C, forms an image I of an object O. Let R be the radius of curvature of the mirror; then it can be shown that

$$\frac{2}{R} = \frac{1}{a} + \frac{1}{b}$$ (i)

in which a and b are the distances of object and image respectively from the mirror as shown in Fig. 162, and L and l are the distances of object and image respectively from the center of curvature C.

From the similar triangles OC and IC, Fig. 162, we have

$$\frac{l}{L} = \frac{I}{O} = \frac{R-b}{a-R}$$ (ii)

in which I and O represent the diameters of image and object respectively.

Eliminating b between equations (i) and (ii) and solving the

resulting equation for R, we have

$$R = \frac{2ax}{x+1} \qquad\qquad \text{(iii)}*$$

where $x = l/L$.

The value of R computed from equation (iii) is not appreciably affected by an error in focusing.

Apparatus. — The unsilvered concave surface of a lens may be used as a mirror if a bright object such as a window is employed.

Work to be done. — Clamp the mirror in a vertical position squarely in front of a window and at a distance of about four meters from the window. Place a vertical millimeter scale between the mirror and the window and adjust the position of the scale until the image of the window is sharply focused on the scale. Measure the distances a, b, l, and L. Any convenient dimension of the window, such, for example, as the length across two window-panes, may be used for L.

Readjust the focus ten or more times and make repeated measurements of b and l.

Repeat the above observations with the mirror placed at a distance of about six meters from the window.

Computations and results. — Compute the value of R by equation (i) and also by equation (iii), using the average of the measured values of b and l for each distance between mirror and window.

EXPERIMENT 115.

INDEX OF REFRACTION OF WATER.

The object of this experiment is to determine the index of refraction of water by a simple method not requiring the use of

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* This equation (iii) results from the solution of a quadratic equation and two values of R are found, namely, $R = a$ and $R = 2ax/(x + 1)$. The former of these two values applies only in case the object is located at the center of curvature of the mirror. In this case the image could also be at the center of curvature, and object and image would be the same in size, making $x = 1$. Equation (iii) is applicable in every case without exception.

the spectroscope. The method here given is essentially that employed by Newton.*

Theory. — Two scales *OM* and *XM*, Fig. 163, are fixed at right angles to each other, and one is adjusted to a horizontal and the other to a vertical position in a vessel which is to contain the water. The eye sights through a narrow slit at *S*, past a horizontal thread at *T*, thus establishing the sight-line *ST*, the readings of which on the two scales *MO* and *MX* are observed. The vessel is then filled with water up to the point *O* and the reading of the refracted sight-line at *Y* is taken. The angle of incidence *i* and the angle of refraction *r* may then be determined by the equations
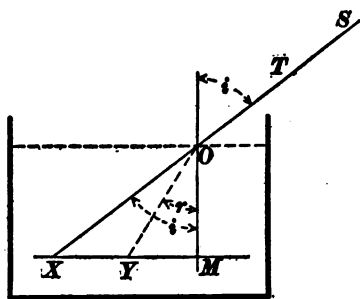
Fig. 163.

$$\tan i = \frac{MX}{MO}$$

$$\tan r = \frac{MY}{MO}$$

and the index of refraction may then be determined from the relation

$$\mu = \frac{\sin i}{\sin r}$$

Work to be done. — Place a large glass jar, preferably a square battery jar, upon a firm support and pour a small quantity of water into it. Clamp two short scales firmly together, setting them at right angles by means of a try-square and record the reading on each scale of the face of the other scale. Place the scales in the jar using the surface of the water as a surface plane by which to make the scales horizontal and vertical respectively.

* See Preston, *The Theory of Light*, third edition, page 49.

The scale OM will be vertical when its reflected image is in line
with it.   Adjust the position of the sight-line ST so that it
crosses the scales at two convenient readings O and X and
record these readings.   Then fill the jar to the level of O and
take the reading $Y.$ In filling the jar to the point O sight
along the surface of the water underneath as if reading a hydrom-
eter scale.   The water to be used in this experiment may be
rendered perfectly clear by adding a trace of alum to it and pre-
cipitating the alum by a few drops of ammonia.

Repeat the above observations using at least five different
values of the ratio MX/MO.   To remove the water from the
jar preparatory to repeating the observations use a siphon.

Computations and results. — From each set of observations
determine values of the angles i and r, and from each pair of
values of i and r compute the value of μ and take the mean
of these values of μ as the final result.

EXPERIMENT 116.

INDEX OF REFRACTION OF GLASS.

The object of this experiment is to determine the index of re-
fraction of glass using very simple apparatus.

Apparatus and method. — A divided circle dd, Fig. 164a,
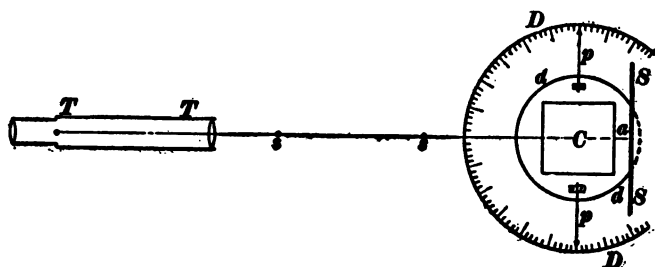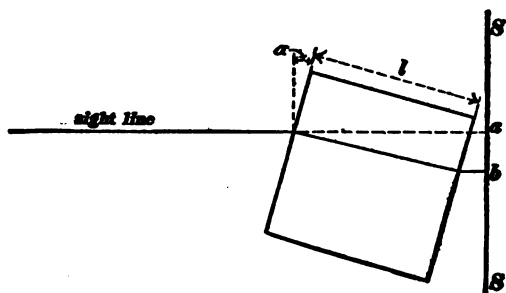

Fig. 164a.

rests flat upon a table, and the small circular table dd turns
upon a pivot which is at the center of the divided circle.   The
small circle dd carries two pointers pp by means of which the

3—4

angular movement of the circle *dd* may be read off on the larger circle. A large glass cube *C,* such as is used for paper-weights, is placed upon the small circle *dd,* and a millimeter scale *SS* with its lines long enough to be seen partly through the cube and partly above the cube is placed as shown in the figure. A reading telescope *TT* is used to establish the sight-line, or a sight-line *ss* may be established by means of a fine slit and vertical thread as in Experiment 115.

Work to be done. — Measure the thickness *l* of the cube from front to back; that is, in the direction of the sight-line in Fig. 164*a.* Turn the glass cube until the divisions of the scale as seen through the cube coincide with the divisions of the scale as seen above the cube and take the readings of both ends of the

Fig. 164b.

pointer *pp.* It is not possible to make the divisions of the scale as seen through the cube coincide throughout with the divisions of the scale as seen above the cube. It is here intended that the coincidence shall take place in the sight-line *ss,* the reading of which on the scale *SS* is represented by *a.*

Turn the glass cube to the right through the angle *a* as shown in Fig. 164*b.* The scale as seen through the cube will then appear to be shifted to the left. Observe the reading *b* of the sight-line on the scale as seen through the cube and read both ends of the pointer.

Turn the cube to the left through an approximately equal

angle and take the reading b' of the sight-line as before, and read both ends of the pointer. These observations should be repeated ten or more times, using a series of values of the angle α.

Computations and results. — In order to calculate the index of refraction of glass from one set of the above observations, find the sine of i and the sine of r, where i is the angle of incidence and r is the angle of refraction. The angle of incidence i is equal to the measured angle α. To calculate the angle of refraction r, let x be the length of the refracted ray in the cube, and let d be the distance ab in Fig. 164. Then

$$d = x \sin (i - r) \tag{i}$$

and

$$l = x \cos r \tag{ii}$$

whence, dividing equation (i) by equation (iii) and developing the expression $\sin (i - r)$ we have :

$$\frac{d}{l} = \frac{\sin i \cos r - \cos i \sin r}{\cos r} \tag{iii}$$

which, when simplified gives :

$$\tan r = \tan i - \frac{d}{l \cos i} \tag{iv}$$

from which r may be determined when d, l and i have been measured. The values of i and r being thus found, the index of refraction may be calculated from the equation

$$\mu = \frac{\sin i}{\sin r} \tag{v}$$

Calculate the value of μ in this way from the mean of the observed values of α and $a - b$ for each of the chosen values of α.

EXPERIMENT 117.

INDEX OF REFRACTION BY THE SPECTROMETER.

The object of this experiment is to determine the index of refraction of a prism of glass.

Apparatus. — The apparatus described in Experiment 6 as the reflecting goniometer is called a spectrometer when a prism is mounted upon the center table and arranged to refract the light from the collimator into the telescope.

Work to be done. (*a*) *Measurement of angle of prism.* — Adjust the spectrometer as explained under Experiment 6, and measure the angle between the refracting faces of the prism.

(*b*) *Measurement of angle of deviation.* — Having adjusted the spectrometer as described in Experiment 6, place a sodium flame in front of the slit *S*, Fig. 165, bring the telescope into the posi-
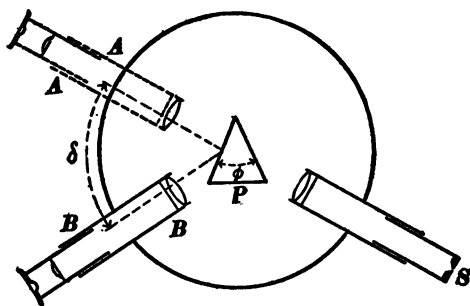
Fig. 165.

tion *AA* so that the cross-wires are coincident with the image of the slit, and take the circle reading. Then put the prism *P* in place, turn the telescope into the position *BB*, turn the prism until the angle δ is at its minimum value, then bring the cross-wires into coincidence with the image of the slit and read the circle. The difference between these two circle readings gives the angle of deviation δ.

If it is desired to determine the index of refraction of the glass for wave-lengths other than that of sodium light, a beam of sun-

light may be thrown through the slit and the cross-wires may be brought into coincidence with any desired Fraunhofer line. In this case the prism P should be adjusted to give minimum deviation of the chosen line. Figure 166 is a diagram of the positions of the principal Fraunhofer lines in the solar spectrum. The letters at the top are the letters originally used by Fraunhofer to

Fig. 166.

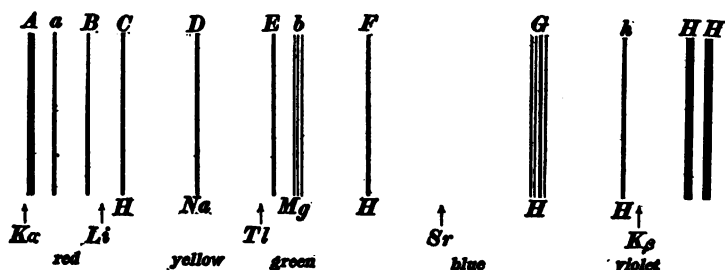designate the more prominent lines, and the letters below show to what chemical elements the various lines belong. The positions of some of the flame spectrum lines of calcium, lithium, thallium, and strontium are also indicated. The wave-lengths corresponding to the various Fraunhofer lines are given in the following table :

TABLE.

WAVE-LENGTHS λ OF LIGHT IN AIR AT 20° C. AND 760 MM.

(Expressed in 0.001 mm.)

The letters in the first column are the customary Fraunhofer names of the lines of the solar spectrum. The values of λ increase about one part in a million for 1° C. rise of temperature or for 3 mm. decrease of pressure.

Name of Line.	Corresponding Chemical Element.	λ	Name of Line.	Corresponding Chemical Element.	λ
A {	—	0.7628	b_1	Mg	0.51837
	—	0.7621	b_2	Mg	0.51727
	—	0.7594	b_3	Fe	0.51690
a	—	0.7185	b_4	Mg, Fe	0.51674
B	O	0.6870	F	H	0.48614
C	H	0.65629	—	Sr	0.46074
D_1	Na	0.58960	(f)	H	0.43405
D_2	Na	0.58900	G	Fe, Ca	0.43079
E {	Fe, Ca	0.52704	—	K, Fe	0.4046
	Fe	0.52696	H	H, Ca	0.3968

Computations and results. — Having measured the angle ϕ of the prism and the angle δ of deviation (see Fig. 165), the index of refraction of the glass for the chosen wave-length is given by the formula

$$\mu = \frac{\sin \frac{1}{2}(\delta + \phi)}{\sin \frac{1}{2}\phi}$$

This equation gives the index of refraction of the glass referred to air. The index of refraction referred to vacuum may be found by multiplying this result by the index of refraction of air referred to vacuum. This, for air at 0°C. and 760 mm. pressure varies from 1.0004 for Fraunhofer's A line to 1.000401 for Fraunhofer's H line.*

EXPERIMENT 118.

INDEX OF REFRACTION BY THE TOTAL REFLECTOMETER.

The object of this experiment is to determine the index of refraction by measuring the angle of total reflection.

Theory — When a beam of light in a dense medium strikes the boundary of the medium beyond which is a rare medium such as

Fig. 167.

air, the beam is totally reflected if the angle of incidence exceeds a certain critical value which is shown in Fig. 167, and which satisfies the following equation
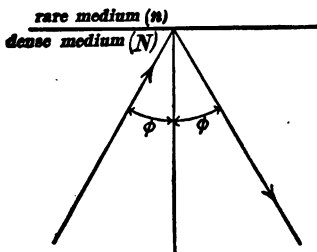
$$n = N \sin \phi \qquad (i)$$

in which n is the index of refraction of the rare medium and N is the index of refraction of the dense medium.

If a piece of glass or crystal having a polished face is submerged in an optically dense † liquid, the surface of the glass or crystal exhibits total reflection when the angle of incidence ex-

* See Landolt and Börnstein, *Physikalische Tabellen*, page 419.
† Having a large index of refraction.

ceeds the critical value ϕ; and if the angle ϕ is determined by observation, the index of refraction N of the dense liquid being known, then the index of refraction of the glass or crystal can be determined from equation (i).

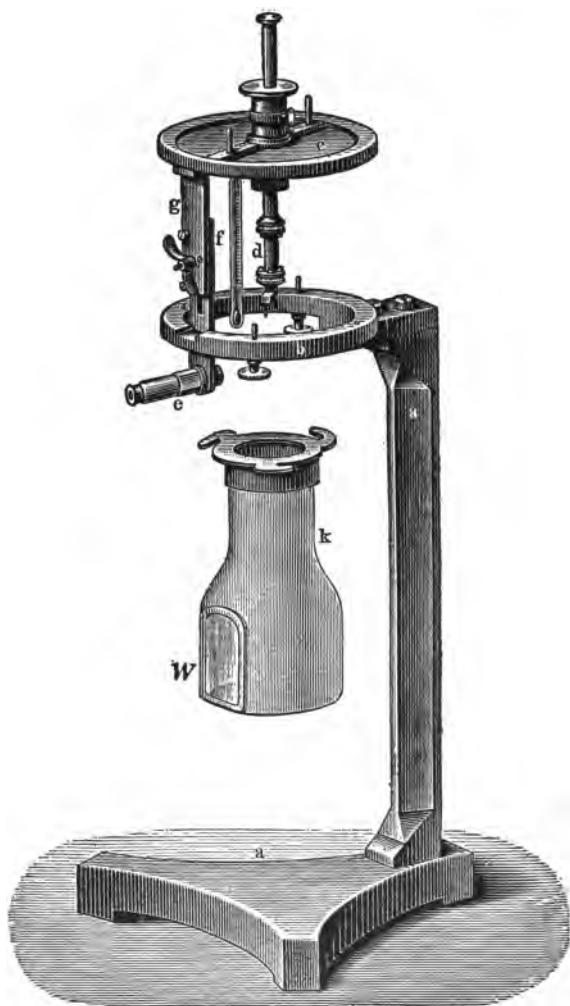**Apparatus.** — An instrument designed for the determination of

Fig. 168.

the index of refraction by the method above outlined is called a *total reflectometer.* A simple form of total reflectometer is shown in Fig. 168. A small piece of the substance of which the index of refraction n is to be determined has a polished reflecting face cut upon it, and it is mounted on the axis d, Fig. 168, so that the axis lies in the plane of this face (the direction of a small face may best be judged by holding a flame so that its reflected image on the small face may be seen).

The substance to be tested having been properly attached to the axis d, the bottle k containing a liquid * of known index of refraction N is brought up into position so that the body is submerged in the liquid and visible through the window w.
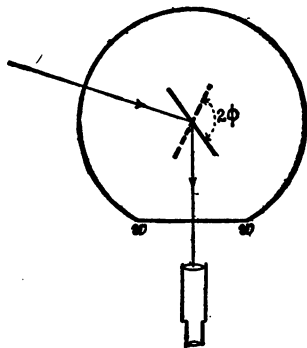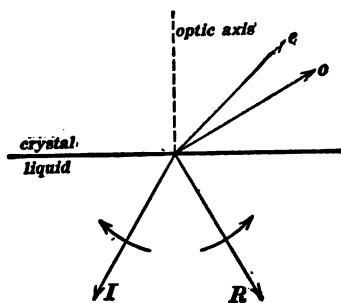
Fig. 169.   Fig. 170.

The small sighting telescope e should be focused for parallel rays and its axis should be parallel to the plane of the divided circle c, or, in other words, at right angles to the axis d. The bottle k is surrounded by a sheet of translucent paper and illuminated at one side by a sodium flame. The axis d is turned until the sharply-defined boundary of total reflection is coincident with the cross-hair of the telescope, and the circle reading is taken. The position of the reflecting surface is shown by the heavy full-line in Fig. 169. The axis is then turned until the reflecting sur-

* The index of refraction of carbon bisulphide is 1.63. The index of refraction of α-mono-bromnaphthalin is 1.66. The index of refraction of methyl iodide is 1.74.

face takes up the position shown by the heavy dotted line in Fig. 169, the boundary of total reflection is again brought into coincidence with the cross-hair of the telescope and the circle reading is again taken.   The difference of these two circle readings gives the angle 2ϕ.

When the substance to be tested is a doubly refracting crystal, there are in general two values of the index of refraction, and consequently two boundaries of total reflection, and therefore two values of the angle ϕ are to be measured.   The indices of refraction of a uni-axial crystal, Iceland spar for example, are best determined on a face which is cut at right angles to the optic axis of the crystal, and it is necessary to distinguish which boundary belongs to the ordinary index of refraction, and which to the extraordinary index of refraction, as follows: Figure 170 represents a reflecting face on a crystal of Iceland spar, the reflecting face being perpendicular to the optic axis.   The incident ray I partly enters the crystal where it forms the ordinary ray' o and the extraordinary ray e, and it is partly reflected as the ray R. Turning the incident ray in the direction of the curved arrow brings the rays o and e more and more nearly parallel to the reflecting face ; the boundary of total reflection corresponding to the ordinary index of refraction occurs when o is parallel to the reflecting face, and the boundary of total reflection corresponding to the extraordinary index of refraction occurs when e is parallel to the reflecting face.   If the incident ray I is polarized with its direction of oscillation perpendicular to the plane of the figure, then the ordinary ray o does not exist in the crystal.   If, however, the incident ray I is polarized with its direction of oscillation perpendicular to the plane of the figure, then the extraordinary ray e does not exist.   Therefore the characteristic features of the boundary of total reflection corresponding to the extraordinary index of refraction will be described on the assumption that the incident ray is polarized and that its direction of oscillation lies in the plane of the figure ; and the characteristic features of the boundary of total reflection corresponding

to the ordinary index of refraction will be based on the assumption that the incident ray I is polarized with its direction of oscillation at right angles to the plane of the figure.

Identification of boundary corresponding to the extraordinary index. — Let rr', Fig. 171, be polarized rays of light with their direction of oscillation lying in the plane of the figure, and let the angles ϕ represent the critical angle of total reflection. The rays r' partly enter the crystal as the rays e and are partly reflected as the rays a'. The
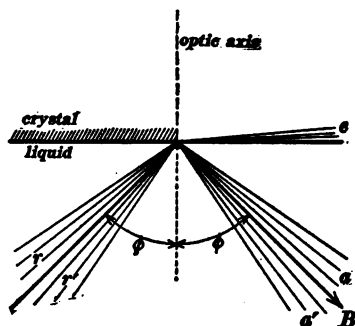

Fig. 171..

rays r, however, are totally reflected as the rays a, and therefore if the rays r and r' are of equal intensity the rays a' will be weaker than the rays a, a sudden change of intensity occurring at the boundary of total reflection B. According to the assumption, the incident rays r and r' are polarized with their direction of oscillation lying in the plane of the figure, and therefore the reflected rays a and a' are polarized with their direction of oscillation lying in the plane of the figure, and inasmuch as, under the assumptions, the rays in the crystal are extraordinary rays, it is evident that the boundary B is that which corresponds to the extraordinary index of refraction, and this boundary can be identified as follows: *Allow the light B to enter the eye through a Nicol prism the longer diagonal of the face of which lies in the plane of the figure, then both sets of rays a and a' are cut out, so that the boundary B disappears from view.*

Identification of boundary corresponding to the ordinary index. — Assuming the incident rays r and r', Fig. 171, to be polarized with their direction of oscillation at right angles to the plane of the figure, then by similar argument to the above it can be shown that *the boundary of total reflection which corresponds to*

the ordinary index of refraction disappears from view when the
light B enters the eye through a Nicol prism the shorter diagonal of
the face of which lies in the plane of the figure.

In the case of Iceland spar the angle ϕ corresponding to the
ordinary index of refraction is greater than that corresponding to
the extraordinary index because the ordinary index of refraction
is greater than the extraordinary index of refraction, but if the
relative magnitudes of the two indices of refraction are not known,
then each boundary of total reflection has to be identified as be-
longing either to the extraordinary index or to the ordinary index,
as here explained.

Work to be done. — Determine by the above method the index
of refraction of several samples of glass, and determine the ordi-
nary and extraordinary indices of refraction of a plate of a doubly
refracting crystal (uni-axial) of which the reflecting face is cut
perpendicular to the optic axis.

EXPERIMENT 119.

STUDY OF REFRACTION AT A PLANE SURFACE.

The object of this experiment is to observe the visual effects
due to the astigmatism which results when spherical waves are
refracted at a plane surface, and to plot the caustic curve of the
refracted wave.

Theory. — Spherical waves emanate from a point O, Fig.
172, in glass or water, and pass into air across the plane surface
AB.   The dotted line WW represents the position an advanc-
ing spherical wave would have reached at a given instant if it had
not passed out of the water into the air, and the line $W'W'$ rep-
resents the actual wave in the air.   This wave $W'W'$ is not
spherical ; that is, refraction at a plane surface is subject to spher-
ical aberration.   Furthermore, a small portion of the wave at aa
has two radii of curvature ; or, in other words, the pencil of rays
near aa is an astigmatic pencil, that is, oblique refraction at a plane
surface is subject to astigmatism.   The intersection of the wave

$W'W'$ with the plane of the paper, Fig. 172, forms at *aa* the arc of a circle whose center is at *C*. Suppose a plane to be passed through the line *DCa*. perpendicular to the plane of the paper. The intersection of the wave $W'W'$ with this plane at *aa* is the arc of a circle whose center is at *D*.

Suppose a person to look at the point *O*, allowing the pencil of rays *aa* to enter the two eyes. If the head is held erect so that the eyes lie in a horizontal line (perpendicular to the plane
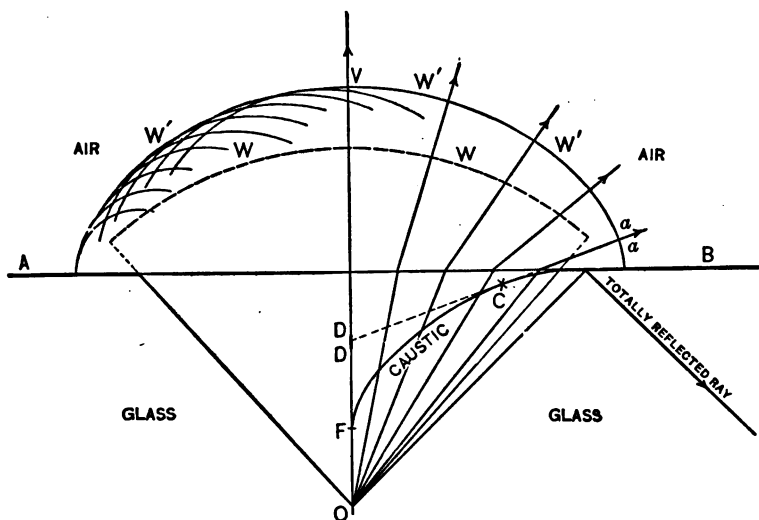

Fig. 172.

of the paper) then the curvature of the portion *a* of the wave front about the vertical axis determines the location of *O* as perceived by the two eyes, and the point *O* appears to be at *D*. If, on the other hand, the head is held on one side so that the two eyes lie in a vertical line (in the plane of the paper) then the curvature of the portion *aa* of the wave front about a horizontal axis determines the location of *O* as perceived by the two eyes, and the point *O* appears to be at *C*. A short vertical line at *D* and a short horizontal line through *C* are the two focal lines of the astigmatic pencil *aa*.

The caustic curve FC, Fig. 172, is the locus of the center of curvature of the wave front $W'W'$, or the caustic curve may be defined as follows: Imagine lines to be drawn normal to the curve $W'W'$ at each point; the envelop of these lines will be the caustic curve.

Work to be done. — Place a vertical scale VO, Fig. 173, and a horizontal scale AB in a vessel of water, the scale AB lying on a level with the surface of the water. The lower end of the scale VO is a very small shallow cup in which a small drop of mercury is placed. The walls of the containing vessel and the lower end of the scale VO are painted black so as to make the drop of mercury clearly visible and to enable one to
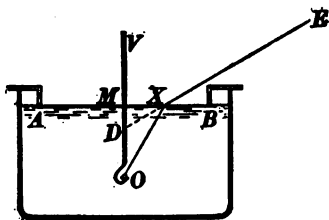

Fig. 173.

see clearly the reflected image of the vertical scale in the water surface. The drop of mercury appears to lie against the face of the reflected image of the upper end of the vertical scale when the line joining the observer's eyes is horizontal, whereas the drop of mercury seems to be very much nearer to the eyes than the reflected image of the vertical scale when the line joining the eyes is vertical.

To determine the caustic curve, proceed as follows: Look at the drop of mercury with one eye placed at E, Fig. 173, and record the reading on the reflected image of the vertical scale of the point D where the mercury drop appears to stand, and also record the reading of the point X where the line of sight crosses the horizontal scale. In making these readings, it is convenient to set a reference line by which the position of the eye may be located. This may be done by clamping a horizontal rod to an adjustable stand and looking at the drop of mercury over the edge of the rod.

Observe the readings X and D for a series of positions of the eye E.

Results. — Make a plot showing the scales OV and AB, Fig. 173, as axes, lay off each pair of observed values of MX and MD and draw each straight line DE. Then draw the caustic curve.

EXPERIMENT 120.

FOCAL LENGTH OF A SIMPLE LENS.

The object of this experiment is to determine the focal length of a simple converging lens.

Theory. — When a beam of parallel rays (parallel to the axis of the lens) passes through a converging lens, the rays are focused at a point distant f from the center of the lens. This point is called the *principal focus* of the lens and the distance f is called the *principal focal length* of the lens.

The principal focal length f, or simply the focal length, of a lens depends upon the material of the lens and the curvature of its two surfaces. In fact, we have

$$ f = \frac{1}{(\mu - 1)\left(\dfrac{1}{R} + \dfrac{1}{R'}\right)} \tag{i}$$

where μ is the index of refraction of the glass of which the lens is made, and R and R' are the radii of curvature of the two surfaces of the lens.

When light from a nearby source A is allowed to pass through a converging lens it is brought to focus at a point B.* The two points A and B are called *conjugate foci* and their distances a and b from the lens satisfy the equation

$$ \frac{1}{f} = \frac{1}{a} + \frac{1}{b} \tag{ii}$$

The focal length of a simple lens may be determined by measuring the distance from the center of the lens to a screen upon which the sun's rays are focused by the lens. A more accurate

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* This statement applies to the case in which the source of light is at a greater distance from the lens than its principal focus.

method for determining the focal length of a lens is to measure the distances *a* and *b* from the center of the lens to a point source *A* and to the point *B* where the light from *A* is brought to a focus by the lens, the principal focus being then calculated with the help of equation (ii).

Apparatus. — The lens *L*, Fig. 174, of which the focal length is to be determined, is mounted on a carriage which slides along a horizontal beam *BB*. At one end of this beam is a lamp *G*
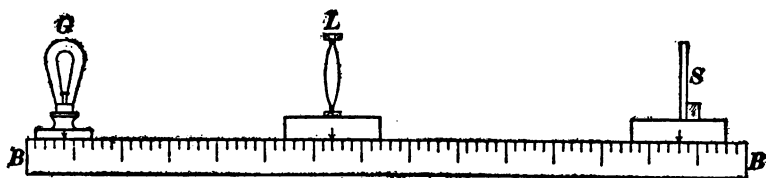

Fig. 174.

and at the other end is a screen *S* upon which an image of the lamp is formed by the lens.

When an incandescent lamp is used in this way, a certain chosen part of the lamp filament must be focused on the screen and the distances *a* and *b* must be measured from the center of the lens to the chosen part of the filament and to the screen respectively. A somewhat better arrangement is to place in front of the lamp a translucent screen upon which a black line or cross is drawn.

Work to be done. — (*a*) Focus the rays of the sun upon a screen by means of the lens, and measure its focal length directly.

(*b*) Place the lens upon the optical bench as shown in Fig. 174, set it at a series of distances *a* from the lamp *G*, focus the chosen portion of the filament upon the screen *S* in each case, and measure the corresponding value of *b*.

(*c*) Measure the radii of curvature of the two surfaces of the lens by means of the spherometer as explained in Experiment 8.

Computations and results. — Calculate the value of *f* from each pair of values *a* and *b* obtained under (*b*) above. The value of *f* calculated from that pair of values of *a* and *b* which are nearly

equal to each other is the most reliable. From this value of f and the values of R and R', calculate the index of refraction of the glass of which the lens is made.

EXPERIMENT 121.

FOCAL LENGTH OF A COMPOUND LENS.

The object of this experiment is to determine the focal length of a compound lens such as a photographic lens.

Theory. — The focal length of a simple lens may be determined by measuring the distances a and b as explained in Experiment 120. In the case of a compound lens, however, it is impossible to measure the distances a and b directly, inasmuch as the position of the center of the lens is indeterminate. To determine the focal length of a compound lens, place the lens on the optical bench, Fig. 174, measure the total distance D from the lamp (chosen part of the filament) to the screen, and measure the distance d between the two positions of the lens which give a sharp image of the lamp on the screen. The two positions of the lens are indicated in Fig. 175, one by full lines, and the other by dotted lines, the lens being represented as a simple lens for the
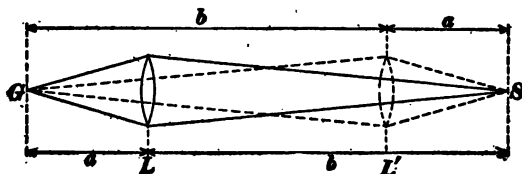

Fig. 175.

sake of clearness. It is evident from Fig. 175 that the distance through which the lens is moved from the position L to the position L' is equal to $b — a$, so that we have the two equations,

$$a + b = D \qquad \text{(i)}$$

$$a - b = d \qquad \text{(ii)}$$

from which a and b may be calculated. The focal length f may then be calculated from equation (ii) in Experiment 120.

Work to be done. — Place the compound lens on the optical bench, Fig. 174, as explained above. Set the lens repeatedly at each position L and L', Fig. 175, measuring the distance d each time. Measure the distance D from the chosen portion of the lamp filament to the screen S.

Computations and results. — From the mean of the measured values of d and D, calculate a and b from equations (i) and (ii) and then calculate the value of f.

EXPERIMENT 122.

STUDY OF SPHERICAL AND CHROMATIC ABERRATION, AND OF ASTIGMATISM.

The object of this experiment is to study the spherical aberration, chromatic aberration, and astigmatism of a simple lens and to determine the magnitude of each of these effects.

Theory. *Spherical aberration.* — A plane or spherical wave after passing through a simple lens is not spherical, and it is therefore not concentrated at a point. Thus WW, Fig. 176, represents the shape (somewhat exaggerated) of a plane wave after it passes through a simple lens. The edge portions of the wave are focused at e and the central portion of the wave is focused at c. This departure of

Fig. 176.

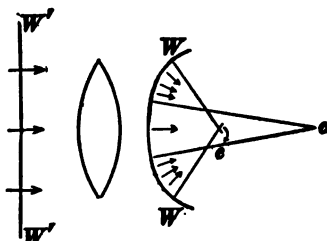a refracted wave from a true spherical shape is called spherical aberration.

Chromatic aberration. — The focal length of a lens depends upon the wave-length of the light which passes through it. Thus, a sharply-defined luminous object like the filament of an incandescent lamp may be focused on a screen by means of a

3—5

simple lens and by adjusting the distance of the screen from the lens, the image of the filament may be made red with a fringe of blue, or blue with a fringe of red. This variation of the focal length of a lens with wave-length of light is called chromatic aberration.

Astigmatism. — A bundle of rays drawn at right angles to a small portion of a wave front is called a pencil of rays. If the small portion of the wave front is a portion of a spherical surface, the rays of the pencil meet at a point (at the center of curvature of the spherical surface) and we have what is called a homocentric pencil of rays. The point at which the rays of a homocentric pencil meet is called the focal point of the pencil. A pencil of parallel rays is a special case of a homocentric pencil with its focal point at infinity.

However complicated the curvature of a wave front may be, a small portion of it is of the shape of a small part of the outer sur-
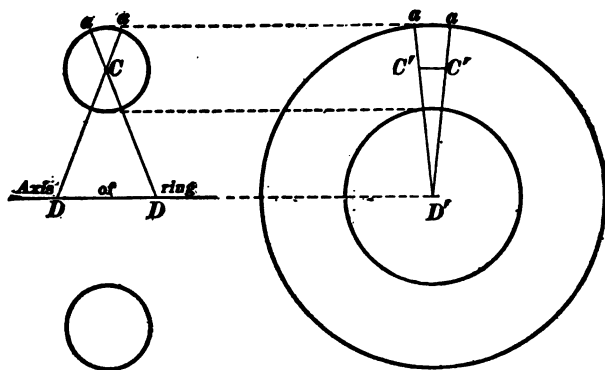

Fig. 177.

face of a ring, having a definite maximum curvature in a certain direction and a definite minimum curvature in a direction at right angles to this. Consider a small portion *aa*, Fig. 177, of the outer surface of a ring. Imagine normals to be drawn from each point of this small portion constituting a pencil of rays. These rays all intersect at *C* along a line perpendicular to the plane

of the figure, and again at DD along the axis of the ring. Such a pencil of rays is called an astigmatic pencil and the two lines C and D are called its focal lines.

A homocentric pencil of rays is converted into an astigmatic pencil when it passes obliquely through a simple lens. Thus the plane wave $W'W'$, Fig. 178, after passing through the lens LL, becomes a wave WW which has two focal lines, one at C and one at D, as shown. The effect of a lens in converting an oblique homocentric pencil into an astigmatic pencil is called astigmatism. A
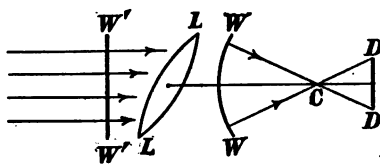

Fig. 178.

lens of which the surfaces are not spherical may have astigmatism for a pencil of rays parallel to its axis. Thus, the astigmatism of the eye is due to the elliptical shape of the surfaces of the cornea and crystalline lens.

Apparatus. — The lens to be studied should be a fairly large simple lens having focal length equal to seven or eight times its diameter, and it should be mounted upon an optical bench, as shown in Fig. 174. To determine the focal length of the lens under any of the following conditions, set the lens at a measured distance a from that part of the lamp filament which is to be used, move the screen S until this part of the filament is sharply focused in red, blue, or white, as the case may be, measure the distance b from the lens to the screen, and calculate f.

Work to be done. *Chromatic aberration.* — Shade the central portion of the lens by means of an opaque circular disk slightly smaller in diameter than the lens, and determine the focal length of the edge portions of the lens for red light and for blue light. In the first case, the screen is to be adjusted until a sharp red image of the selected portion of the lamp filament is formed on the screen ; this image will be fringed with blue. In the second case, the screen is to be adjusted until a blue image of the lamp filament is formed ; this image will be fringed with red. The

central portion of the lens is screened so as to eliminate compli-
cations due to spherical aberration. The difference between the
focal length for red light and the focal length for blue light
divided by the mean focal length is a measure of the chromatic
aberration.

Spherical aberration. — Shade the central portion of the lens
and determine the focal length of the outer portion of the lens
for, say, red light. Shade the outer portion of the lens by means
of an opaque screen with a small hole through it, and determine
the focal length of the central portion of the lens, also for red
light. The difference between these focal lengths divided by their
mean is a measure of the spherical aberration of the lens.

Astigmatism. — In the above work, and in Experiments 120
and 121, the lens is set so that the line drawn from object to
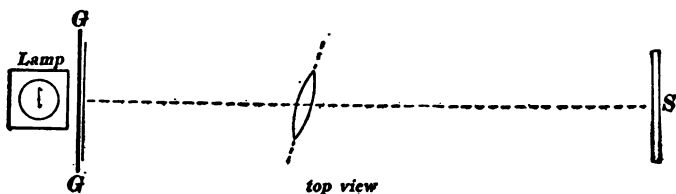image crosses both surfaces of the lens at right angles. When

top view

Fig. 179.

the lens is perceptibly inclined to this line, a pencil of rays from
a point of the object becomes an astigmatic pencil of rays
after passing through the lens, and the point of the object is
focused by the lens first as a short vertical line and then, at a
slightly greater distance from the lens, as a short horizontal line.
The result is that the focal length of the lens for vertical lines in
the object is different from the focal length of the lens for hori-
zontal lines in the object.

Place a plate of ground glass *GG*, Fig. 179, in front of the
lamp, and stretch a horizontal and a vertical wire across the face
of the ground glass plate. Set the lens obliquely as shown in the
figure, shade the outer portion of the lens, adjust the screen until
the vertical line is in focus, and calculate the corresponding focal

length.   Then adjust the screen until the horizontal wire is in focus and calculate the corresponding focal length.

EXPERIMENT 123.

MAGNIFYING POWER OF A SIMPLE MICROSCOPE.

The object of this experiment is to measure directly the magnifying power of a simple microscope and to compare the result with the value of the magnifying power computed from the focal length of the lens.

Theory. — The simple microscope, or magnifying glass, is a converging lens which is held near the eye and the object to be examined is moved up until it is clearly defined.   The eye is then looking at an enlarged virtual image of the object, which
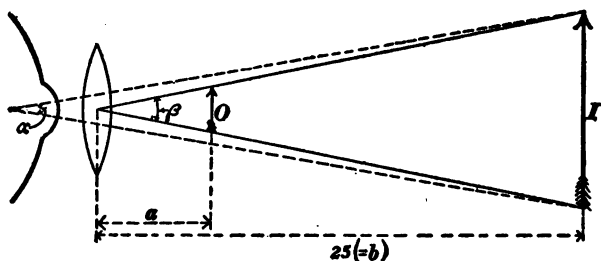

Fig. 180.

image is at the distance of most distinct vision from the eye, usually about 25 centimeters, as shown in Fig. 180.

The magnifying power of a microscope, simple or compound, is defined as the ratio of the apparent size of an object as seen with the microscope to its apparent size as seen with the naked eye at a distance of 25 centimeters, apparent size being in each case measured by visual angle.   Thus, if an object one milli-meter in length be viewed through the microscope with one eye, and a millimeter scale at a distance of 25 centimeters be viewed directly with the other eye, then the magnified image of the object will be seen superposed upon the scale and will appear to be *m*

millimeters long, where m is the magnifying power of the microscope.

The magnifying power of a simple microscope, when the eye is accommodated for a distance of 25 centimeters, is

$$m = \frac{25}{f} + 1 \qquad\qquad \text{(i)}$$

in which m is the magnifying power, and f is the focal length of the lens in centimeters. This equation may be made evident as follows : Ignoring the distance between the lens and the eye, the angle subtended by the object O, Fig. 180, at its distance a from the lens, the angle subtended by the image I at its distance of 25 centimeters from the lens, and the angle subtended by the image at its distance of approximately 25 centimeters from the eye are all equal, and this angle is larger than the angle which would be subtended by the object at a distance of 25 centimeters in the ratio of 25 to a, so that the magnifying power of a simple microscope is

$$m = \frac{25}{a} \qquad\qquad \text{(ii)}$$

Now the object and image constitute a pair of conjugate foci, and, inasmuch as the image is virtual, we have the equation

$$\frac{1}{f} = \frac{1}{a} - \frac{1}{25} \qquad\qquad \text{(iii)}$$

Substituting the value of a from equation (iii) in equation (ii) we have equation (i).

Work to be done. — Mount the magnifying glass on a clamp stand in front of a vertical millimeter scale A, and set up a second vertical millimeter scale B at a distance of 25 centimeters from the lens. Place the right eye near the lens and adjust the position of scale A until its image is seen superposed upon scale B (scale B being viewed with the naked left eye during this adjustment). Then read from scale B the number of millimeters covered by the magnified image of one millimeter of scale A.

This result is the magnifying power, since in this case the eye is accommodated for a distance of 25 centimeters.

Measure the focal length at the lens used, that is, measure the distance between the center of the lens and a screen upon which the sun's light or the image of a distant window is sharply defined.

Computations and results. — From the measured value of the focal length of the lens, compute the magnifying power and compare this with the value of the magnifying power directly determined.

EXPERIMENT 124.

MAGNIFYING POWER OF A COMPOUND MICROSCOPE.

The object of this experiment is to determine the magnifying power of a compound microscope for various lengths of tube.

Theory. — The compound microscope consists essentially of a lens called the *object-glass* which forms an enlarged real image of an object and a magnifying glass or *eye-piece* for viewing this image.

The magnifying power m of a compound microscope is given by the equation

$$m = \frac{b}{a}\left(\frac{25}{f} + 1\right) \tag{1}$$

in which b and a are the distances of image and object respectively from the center of the object glass, and $\left(\frac{25}{f} + 1\right)$ is the magnifying power of the eye-piece. This equation is at once evident when we consider that the image is b/a times as large as the object and that the image is magnified $\left(\frac{25}{f} + 1\right)$ times by the eye-piece.

This equation shows that the magnifying power of a given compound microscope may be increased by lengthening the tube of the instrument, inasmuch as this increases b and decreases a. A good microscope, however, has its lenses designed to give the best

definition for a certain length of tube and to change the tube
length spoils the definition.

Work to be done. — Examine the microscope and identify the
parts.   Look through the microscope with one eye at an object
of known diameter d, a fine wire for example, with the other
eye look at a scale distant 25 centimeters from the eye, and read
on this scale the apparent diameter D of the object seen in the
microscope.   The magnifying power is then D/d.   Measure the
diameter of the wire d by means of a micrometer caliper.

Determine the magnifying power of the microscope for several
lengths of tube, measuring each length.

Computations and results. — Plot a curve showing the relation
between length of tube and the magnifying power of the micro-
scope.

EXPERIMENT 125.

MAGNIFYING POWER OF A TELESCOPE.

The object of this experiment is to determine the magnifying
power of a telescope directly, and also by computation from the
focal lengths of its object-glass and eye-piece.

Theory. — The magnifying power of a telescope is the ratio of
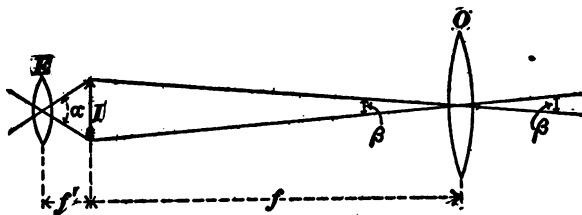the apparent size of a distant object as seen through the telescope

Fig. 181.

to its apparent size as seen with the naked eye, apparent size
being in each case measured by visual angle.

Figure 181 shows the object-glass O and the eye-piece E of
a simple telescope.   Lines drawn from the extremities of the ob-
ject through the center of the object-glass pass through the ex-

tremities of the image I, as shown.   The angle β which is
subtended by I as seen from the center of the object-glass is
the angle subtended by the object as seen directly with the naked
eye, and it measures the apparent size of the object as seen with
the naked eye.   The angle α, on the other hand, measures the
visual angle of the object as seen through the telescope, so that
the ratio α /β is the magnifying power of the telescope ; and this
ratio is sensibly equal to the inverse ratio of the two distances f
and f' which are the focal lengths of the object-glass and eye-
piece respectively.   Therefore the magnifying power of the tele-
scope is given by the equation

$$m = \frac{f}{f'} \tag{i}$$

Work to be done. — Select a telescope having a simple eye-
piece * and clamp it horizontally in a supporting stand.   At a
distance of ten or more meters from the telescope, set up a vertical
meter scale † on which are two markers, each of which may be slid
up and down.   One observer then looks through the telescope
at the middle centimeter of the scale with one eye and directly at
the scale with the other eye, and a second observer adjusts the
markers until their positions as seen by the naked eye of the first
observer coincide with the boundary marks of the centimeter as
seen through the telescope.   The distance between the markers
in centimeters is the required magnifying power of the telescope.

Remove the object-glass and eye-piece and determine their
focal lengths by measuring the distance from each lens to a
screen upon which it focuses the sun's rays.

Computations and results. — Compute the magnifying power of
the telescope from the observed focal lengths and compare this

* An ordinary spy-glass may be used by removing all the lenses except the object-
glass and the single lens next to the eye.

† If it is not convenient to use a scale and markers, a distant brick wall may be
used, the observer counting with the naked eye the number of bricks covered by the
image of a single brick as seen through the telescope.

result with the value of the magnifying power as measured directly.

Note. — The magnifying power of a telescope for an adjacent object is greater than its magnifying power for a distant object because the distance *OI*, Fig. 181, is greater than *f* unless the object is at a very great distance from the telescope. This source of error may be eliminated in the above observations by multiplying the observed magnifying power by *f/b*, where *b* is found from the equation

$$\frac{1}{f} = \frac{1}{a} + \frac{1}{b} \tag{ii}$$

in which *a* is the distance of the object from the object glass.

EXPERIMENT 126.

SPECTRUM ANALYSIS.

The object of this experiment is to familiarize the student with the use of the spectroscope.

Apparatus. — The essential features of the spectroscope as it is usually arranged for spectrum analysis are shown in Fig. 182.
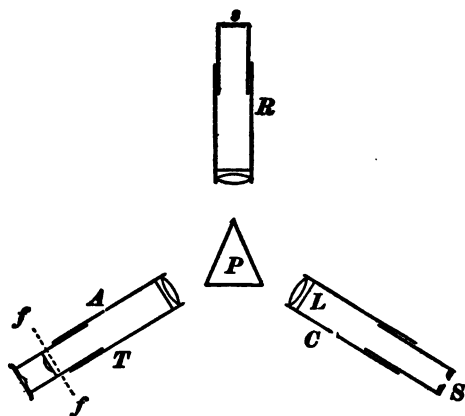

Fig. 182.

The light to be analyzed passes through the slit *S*, through the collimating lens *L*, through the prism *P*, and then to the tele-

scope T where it is focused in the focal plane ff as a spectrum. A transparent scale s is mounted on an auxiliary collimator R, and light from a lamp passes through s, is reflected from the face of the prism P into the telescope, and forms an image of the transparent scale s in the focal plane ff. By this means the positions of the lines in the spectrum can be read off on the scale.

Adjustment of apparatus. — The spectroscope is to be adjusted as follows, the order of the operations being especially important.

1. The slit must be at the principal focus of the collimating lens L. The proper position of the slit is usually marked on the sliding tube which carries the slit. When the proper position of the slit is not so marked, the telescope T must be focused on a distant object, a sodium flame placed in front of the slit, and the sliding tube of the collimator C adjusted until a sharp image of the slit is seen in the telescope.

2. The prism must be set at minimum deviation. This adjust-ment is usually ready-made in a chemical spectroscope. If not, illuminate the slit with sodium light, catch the refracted beam from the prism with the naked eye so as to be able the more easily to sight the telescope properly, then look through the tele-scope and adjust slightly until the image of the slit is visible. Then turn the prism until the image of the slit is farthest towards the side A in Fig. 181, following the image, if necessary, by moving the telescope. The prism is to be clamped in the posi-tion so found.

3. The reflected image of the scale from the auxiliary collimator R must be sharply in focus. Illuminate the scale by a lamp placed about 20 centimeters from it, turn the tube R on its pivot-support until the image of the scale is seen in the telescope, and then adjust the slider of the collimator R until the scale is sharply in focus. The images of slit and scale should not alter their relative positions when the eye is moved sidewise before the eye-piece of the telescope.

4. The sodium line should be made to fall upon a particular division of the scale. The arbitrary scale adopted by Bunsen and

Kirchhoff is almost universally used in chemical spectroscopes, and in this scale the sodium line should fall on 50. This adjustment is made by turning the tube R on its pivot. The tube R should be clamped when this adjustment is made.

Valuation of the scale. — The scale readings of the spectral lines of the different chemical elements must be once for all determined, together with their relative brightness and width or sharpness. The following table gives these data for the Bunsen and Kirchhoff scale.

TABLE.

LINES OF THE FLAME-SPECTRA OF K, NA, LI, CA, SR, AND BA.

According to Bunsen and Kirchhoff's Scale. Sodium Line at 50, Width of Slit One Division.

The column headed P gives the position of the middle of the line.

The column headed B indicates the brightness ; I being the brightest.

The column headed W gives the width of the lines in divisions when the width exceeds one division.

The letter S signifies that the line is sharp and clearly defined, s that it is fairly so, the remaining lines being nebulous and ill-defined.

The most characteristic lines of each element are printed in bold-face type.

The indicated brightness of the lines of Ca, Sr and Ba are the lines of the steady spectrum. If the chlorides are used the lines are at first much brighter.

The colors of the spectrum are approximately — red to 48, yellow to 52, green to 80, blue to 120, and violet beyond.

K			Na			Li			Ca			Sr			Ba		
P	B	W	P	B	W	P	B	W	P	B	W	P	B	W	P	B	W
17.5	Il	s				32.0	I	S	33.1	IV	2	29.8	III				
Faint continuous spectrum from 55 to 120			50.0	I	S	45.2	IV	s	36.7	IV		32.1	II				
									41.7	I	1.5	33.8	II		35.2	IV	2
									46.8	III	2	36.3	II		41.5	III	3
									49.0	III		39.0	III		45.6	III	1.5
									52.8	IV		41.8	III		52.1	IV	
									54.9	IV		45.8	I		56.0	III	2
									60.8	I	1.5				60.8	II	s
									68.0	IV	2				66.5	III	3
															71.4	III	3
															76.8	III	2
												105.0	III	S	82.7	IV	4
153.0									135.0	IV	S				89.3	III	2

The scale of a given instrument can be reduced to the Bunsen and Kirchhoff scale as follows : Observe the readings on the scale of the given instrument of a few known lines distributed throughout the spectrum. Plot a curve of which the abscissas represent these instrument readings and of which the ordinates represent the corresponding Bunsen and Kirchhoff readings. This curve will then give the Bunsen and Kirchhoff reading corresponding to any given reading on the scale of the instrument.

The analysis. — The substances to be examined are vaporized off a small platinum wire held in the lower front (facing the slit) part of a Bunsen flame. The light from the flame passes into the slit of the spectroscope and the position, relative brightness, and width of each line are observed and recorded as in the following table. The presence of one or more of the various elements is established by identifying the lines of its spectrum among the lines so observed.

Examine the spectrum first with narrow slit for bright lines, then with a wider slit for the lines of feeble brightness.

EXPERIMENT 127.

DETERMINATION OF WAVE-LENGTH OF LIGHT BY THE DIF-FRACTION GRATING.

The object of this experiment is to determine the wave-length of the light corresponding to a chosen spectrum line, for example, the wave-length of sodium light.

Theory. — The diffraction grating consists of a large number of equidistant parallel slits in an opaque screen.* The action of the grating is

Fig. 183.

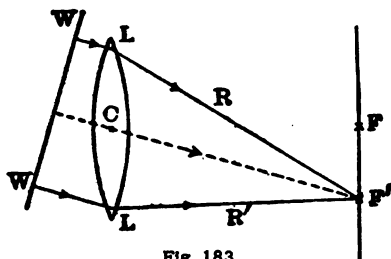most easily described in combination with a lens, and in this case it is necessary to keep in mind the fact that a plane wave

* This statement and the following discussion refers to the transmission grating.

WW, Fig. 183, approaching the lens *LL* obliquely as shown, is focused at the point *F'* in the focal plane of the lens ; the line *F'C* being perpendicular to the wave front *WW*, and since the wave *WW* is focused at *F'* every portion of *WW* must reach *F'* at the same instant ; that is, all rays, such as *R, R'*, etc., drawn from *WW* to *F'* have the same optical length.

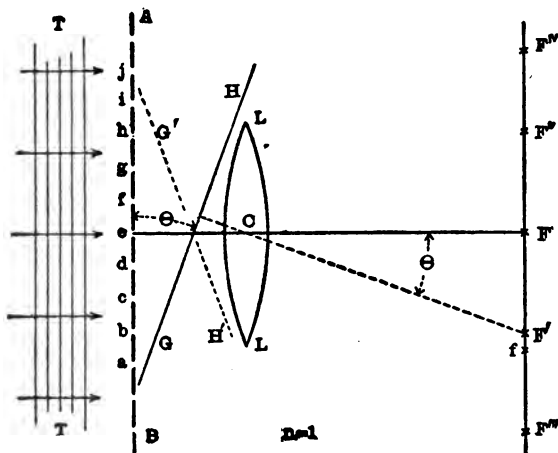A simple train of waves *TT*, Fig. 184, of wave-length λ, approaches a diffraction grating *AB*, as shown. Beyond the

Fig. 184.

grating is a lens *LL* of which the focal plane is represented by the line *F''' F^iv*. The portion of the wave train *TT* which passes through the grating is focused by the lens at a series of points *F, F', F'', F''', F^iv*, etc. This may be explained and the relation between the angle *θ* and the wave-length may be derived as follows : As each wave of the train *TT* strikes the grating, wavelets pass out simultaneously from all the slits, and these wavelets are always in like phase along the line (plane) *GH*, which is *n* full wave-lengths from slit *a*, 2*n* full wave-lengths from slit *b*, 3*n* full wave-lengths from slit *c*, 4*n* full wave-lengths from slit *d*, etc., *n* being any whole number. Furthermore, the point *F'* is at the same distance optically from every point

in the line *GH*. Therefore wavelets from all the slits reach *F'* in like phase; and the wavelets therefore work together to produce brilliant illumination at *F'*. Let *l* be the distance between centers of adjacent slits, then the distance from the slit next below *a* to slit *d* is $4l$ and the distance from slit *d* to the line *GH* is $4n\lambda$, as above explained, so that the angle θ between *AB* and *GH* is such that

$$\sin \theta = \frac{4n\lambda}{4l} \qquad (i)$$

The above argument applies unchanged to the line (plane) *G'H'* and to the point *F''*. Furthermore, the lines *GH* and *G'H'*, and the points *F'* and *F''* in the figure are drawn for the case $n = 1$. For $n = 2$ we have the two luminous points *F'''* and *F^{iv}*. For $n = 3$ we have an additional pair of luminous points still farther from the central point *F*, and so on. The locations of the points *F'*, *F''*, *F'''*, *F^{iv}*, etc., are determined by the value of the angle θ which satisfy equation (i).

It is instructive to show why it is that with a wave train *TT* of given wave-length, the focal plane of the lens is illuminated only at the points *F*, *F'*, *F''*, *F'''*, etc., and nowhere else. Figure 185 shows the line *GH* drawn in the position which satisfies equation (i), and it shows the wavelets from the various slits with their crests all touching the line *GH* at a given instant. Imagine the angle θ to be slightly increased so that *GH* becomes tangent to wavelet *W'* from slit *j* instead of being tangent to wavelet *W*, as shown. Corresponding

Fig. 185.

to this new value of θ and new position of *GH*, we have the new point *f* as shown in Fig. 184, and the relative phases of the various wavelets when they reach *f* are the same as their relative
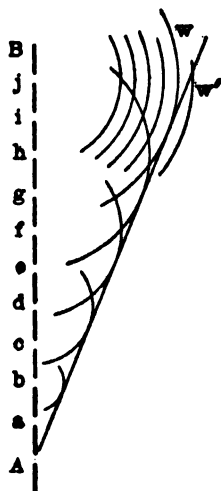
phases when they reach the new position of GH, and it is evident from Fig. 185 that the phases of the wavelets along the new line GH have every possible value ranging from a crest at W' to a hollow of the wavelet from slit e, and again to a crest of the wavelet from the slit next below a.   Therefore the wavelets annul each other at the point f, and the illumination at f is zero. In general, any point f for which the angle θ does not satisfy equation (i) has either zero illumination or a negligible degree of illumination if the grating has a great many slits.

The action of the grating in Fig. 184 may be made more clearly evident if one constructs the nth wavelet from slit a, the $2n$th wavelet from slit b, the $3n$th wavelet from slit c, the $4n$th wavelet from slit d, etc.   The envelop of these wavelets is an actual wave front GH.

Apparatus and work to be done. — The telescope of a spectrometer is focused for parallel rays, the collimator and telescope are then brought into line, and the sliding tube of the collimator is adjusted until the slit is sharply in focus in the telescope.   The diffraction grating GG is then placed upon the spectrometer as
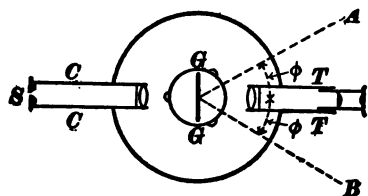
Fig. 186.

shown in Fig. 186, the lines of the grating being parallel with the slit, and the face of the grating being at right angles to the axis of the collimator CC.

Pass sodium light through the slit S, place the telescope in the position shown in Fig. 186, bring the cross-hairs into exact coincidence with the central image of the slit, and read the circle. Then turn the telescope slowly to one side (position A in Fig. 186) until the first-order image of the slit coincides with the cross-hairs, and again take the circle readings.   Then turn the telescope to the other side (position B in Fig. 186) until the other first-order image of the slit is coincident with the cross-hairs, and again take the circle readings.   From the mean value

of the angle ϕ so determined, the wave-length of the light may be calculated from the equation

$$\lambda = l \sin \phi \qquad \text{(ii)}$$

If the telescope is turned until the nth-order image of the slit is coincident with the cross-hairs and the corresponding angle ϕ_n is observed, then the wave-length is determined by the equation

$$\lambda = \frac{l}{n}\sin \phi_n \ . \qquad \text{(iii)}$$

Determine in this way the wave-length of sodium light and of the light corresponding to the prominent Fraunhofer lines in the solar spectrum.

Consult the instructor concerning the value of the grating space l.

Results. — Tabulate the values of the wave-lengths of the various lines of the solar spectrum and compare these values with the values given on page 37.

Note. — The wave-lengths of many spectral lines have been determined with extreme accuracy by Roland and others. If, therefore, the grating space l is unknown, it may readily be determined from equation (ii) above, the angle ϕ having been measured for a spectral line of known wave-length.

EXPERIMENT 128.

DETERMINATION OF WAVE-LENGTH BY THE INTERFEROMETER.

The object of this experiment is to determine the wave-length of light by Michelson's interferometer.

Theory. — The essential features of the interferometer are shown in Fig. 187. Homogeneous light TT from a sodium flame, for example, strikes a half-silvered * glass plate AA and is partly reflected and partly transmitted. The part that is reflected strikes the silvered mirror D, is thrown back, and a

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* A glass plate with an extremely thin coating of silver so as to approximately reflect half the light and transmit half the light falling upon it.

3—6

portion of it passes through *A* and enters the telescope *P* as the wave train *T'*.

The part of the original light that passes through *A* strikes the silvered mirror *C*, is thrown back, and a portion of it is reflected from *A* and enters the telescope *P* as the wave train *T''*.

The light *T'* has traversed the glass plate *A* three times, and the light *T''* has traversed *A* but once.   Therefore a glass
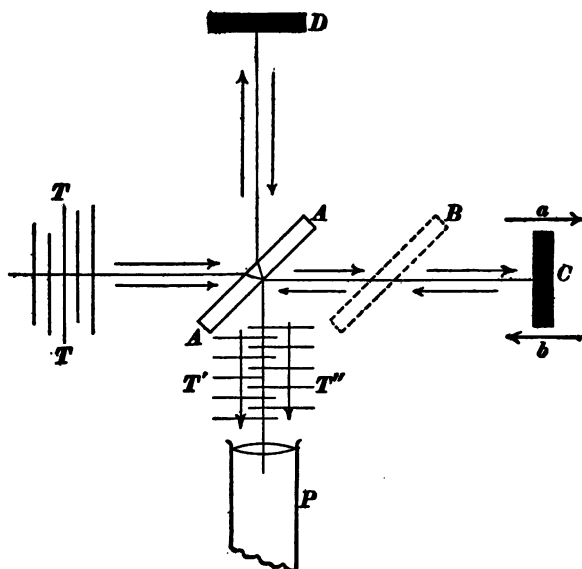

Fig. 187.

plate *B* similar to *A* is placed between *A* and *C* so that the light *T''* passes through the plate *A* once and through the plate *B* twice, thus making the total length of path in glass the same for *T'* as for *T''*.

The light *T'* has been reflected by *A* internally (without change of phase) and the light *T''* has been reflected by *A* externally (with change of phase).   Therefore if the distance 2*AD* is equal to the distance 2*AC*, or if the distance 2*AD* differs from the distance 2*AC* by a whole number of wave-lengths,

the wave trains T' and T'' will correspond crest to hollow so that the center of the field of the observing telescope will be dark. If the distance $2AD$ differs from the distance $2AC$ by an odd number of half wave-lengths, and two wave trains T' and T'' will correspond crest to crest so that the center of the field of the observing telescope will appear bright.

The mirror C is mounted on a sliding carriage actuated by a micrometer screw so as to be moved at will in the direction of the arrow a or in the direction of the arrow b. If the mirror C is adjusted until the center of the field of the telescope is dark, then as the mirror is moved, the center of the field becomes alternately light and dark, and the distance that the mirror is moved is equal to $n\lambda/2$ where n is the number of times that the field has become dark during the movement.

If the distance moved by C is known, as, for example, in terms of the reading of the micrometer screw, the value of λ may be determined, or if the value of λ is known, the movement of the mirror may be determined and the micrometer screw standardized.

The field of view in the telescope when the mirrors C and D are exactly at right angles shows a series of concentric circular interference fringes, light and dark. If the mirrors C and D are not exactly at right angles, a series of approximately straight interference fringes will cross the field of view of the telescope. These bands move sidewise across the field of view when the mirror C is moved in the direction of a or b, and the movement of the mirror is equal to $n\lambda/2$, where n is the number of dark bands which pass by a given point of the field during the movement.

Work to be done. — Use a sodium flame and a collimating lens for producing the train of plane waves TT, and with the interferometer in adjustment, count the fringes which move past the center of the field of the telescope for every fifth of a turn of the micrometer screw which moves the mirror C.

If the interferometer is not in adjustment, see instructor.

Computations and results. — Calculate the wave-length of sodium light from the above observations, knowing the pitch of the screw.

The wave-length of sodium light is known with a high degree of precision (see page 37) so that the above observations furnish a basis for the accurate standardization and calibration of the micrometer screw. Plot a curve of which the abscissas represent the readings of the micrometer screw, and of which the ordinates represent the numbers of interference fringes which have moved past the center of the field of the telescope. These ordinates represent actual distances moved by the mirror C in terms of the half wave-length of sodium light (see page 37).

EXPERIMENT 129.

SACCHARIMETRY.

The object of this experiment is to determine the sugar content of a sample of syrup by observing the rotation of the plane of polarization of polarized light.

Theory. — A substance which rotates the plane of vibration of polarized light is said to be *optically active*. The substance is said to produce right-handed rotation when the plane of vibration is rotated clockwise as seen by the eye receiving the beam of light.

The angle of rotation produced by unit length of the active substance is called its *specific rotation*. The specific rotation of a given substance varies greatly with the wave-length of the light, and it is usually greater for short wave-lengths than for long wave-lengths. Ordinary sugar (cane sugar) is optically active, and for a given wave-length the angle of rotation β of the plane of vibration is proportional to the length l of the solution, and approximately proportional to the number of grams of sugar s per cubic centimeter of the solution so that we may write

$$\beta = ksl \qquad\qquad\qquad (\text{i})$$

in which k is the specific rotation of sugar.   The value of k for cane sugar and for sodium light is 6.65 when l is expressed in centimeters, s in grams per cubic centimeter, and β in degrees.   Therefore, from equation (i) we have

$$s = 0.1504\frac{\beta}{l} \tag{ii}$$

from which the sugar content s of a solution may be calculated when the angle β of rotation of the plane of vibration of polarized sodium light which is produced by l centimeters of the solution has been observed.

Figure 188 shows the essential features of the simple saccharimeter of Mitscherlich ; S is a sodium flame, P is a stationary
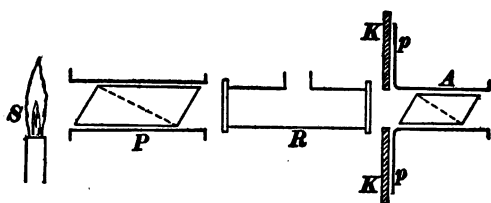
Fig. 188.

Nicol prism (the polarizer), R is the receiving tube for the sugar solution with glass windows at its ends, and A is a rotating Nicol prism (the analyzer) the position of which can be read off the divided circle K by means of the two pointers pp.

To use this saccharimeter, the analyzer A is turned until the field is dark and the circle readings are taken.   The sugar solution is then put in place and the analyzer is again turned to give extinction of light, and the circle readings are again taken.   The difference of these two circle readings gives the value of β, and the length of the receiving tube R inside of the glass windows is the value of l in equation (ii).

The saccharimeter of Mitscherlich cannot be made to give accurate results because of the difficulty of setting the Nicol prism A to give complete extinction ; such a setting cannot be made

accurately.   The instrument to be used in the laboratory is the
half-shade saccharimeter of Laurent, the essential features of
which are shown in Fig. 189.   Light from a sodium flame S
passes through the polarizer P and through a round hole in a
metal plate DD.   One half of this hole is covered by a thin plate
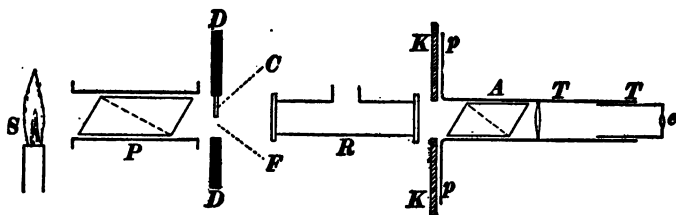
Fig. 189.

of a doubly-refracting crystal cut parallel to the optic axis of the
crystal.   The polarized light then passes through the sugar solu-
tion R, through the analyzing Nicol prism A, and through a
small telescope TT.   The telescope TT is focused on the hole
in the metal plate DD, so that the observer, looking in at e,
sees a round field of light divided sharply into two parts C and F.

The crystal plate C is of such thickness as to produce with
the sodium light one half of a wave-length of relative retardation
between the ordinary and extraordinary rays.   The direction of
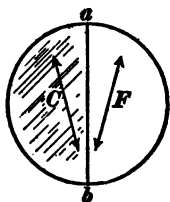
Fig. 190.
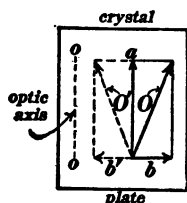
Fig. 191.

oscillation of the polarized light from P is shown by the arrow
F in Fig. 190, being very slightly inclined to the edge ab of
the crystal plate; the optic axis of the crystal plate is parallel to
the edge ab;* and the polarized light, after passing through the

* The optic axis may lie in any direction in the plane of Fig. 190; it is here
assumed to be parallel with the edge ab for the sake of definiteness.

crystal plate, oscillates in the direction of the line C, Fig. 190. The action of the crystal plate is shown in Fig. 191. The oscillations of the incident light are shown by the line O; these oscillations are resolved into the two oscillations a and b; one of these oscillations b is reversed with respect to the other by the different retardations of the ordinary and extraordinary rays in the crystal; and the light, as it emerges from the crystal plate, is composed of the two oscillations a and b' which coalesce to form the resultant oscillation O'.

The analyzing Nicol A, Fig. 189, can of course be turned to give total extinction of light in either portion C or F of the field of view, and there are two positions of the analyzer A at right angles to each other which give equal intensities of illumination over the whole field of view (C and F). For one of these positions, which we will call position No. 1, both portions of the field are very dim; and for the other, which we will call position No. 2, both portions of the field are quite bright. The setting of the analyzer to position No. 1 to give equal luminosities of the portions C and F of the field of view can be made with great accuracy.

Work to be done. — Set up the instrument in a dark corner of the laboratory, placing a dark screen behind the sodium lamp so as to cut out daylight as much as possible, and focus the telescope on the hole in the metal plate DD by adjusting the sliding tube which carries the eye-piece.

1. (a) It is desirable to study the accuracy of the settings of the half-shade saccharimeter as compared with the settings of the saccharimeter of Mitscherlich. Using sodium light, take ten settings of the analyzing nicol prism A to give complete extinction of light in either portion, C or F of the field of view, and determine the probable error of one of these settings. (b) Take ten settings of the analyzer to position No. 1, as above explained, and determine the probable error of one of these settings. (c) Take ten settings of the analyzer to position No. 2, as above explained, and determine the probable error of one of these settings.

2. (*a*) Using sodium light, take five settings of position No. 1 of the analyzer before the sugar solution is in place, and record the circle readings. Repeat with analyzer turned through 180°.

(*b*) Place the sugar solution in position, having measured the length of the tube between the glass windows, re-focus the telescope on *DD*, and, using sodium light, take five settings of position No. 1 of the analyzer. Repeat with the analyzer turned through 180°.

3. The direction of rotation of the plane of vibration of the light by the sugar solution may be a question, and the following test must be made to determine it. Remove the sugar solution, and set the analyzer so as to extinguish the light in the half-field *F*, using daylight (it will be noted that the light which is passed through the crystal plate *C* cannot be extinguished if daylight is used). Then replace the sugar solution, and the *direction of rotation of the plane of vibration is the direction in which the analyzer must be turned to give with daylight the following succession of colors: light blue, deep blue, purple, orange, yellow.**

4. It is impossible to tell from the observations taken under (2) whether the angle of rotation of the plane of vibration is greater or less than 180°. In order to establish this point, repeat the observations (2) using daylight which has passed through ruby glass. The value of β in this case is about three fourths as great as with sodium light.

Computations and results. — Let *O* and *O'*, Fig. 192, be the circle readings obtained under 2 (*a*) above and let *S* and *S'* be the circle readings obtained under 2 (*b*) above. Then if the sugar solution has turned the plane of polarization to the right (clock-

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* The explanation of this succession of colors is as follows : Sugar solutions, right-handed or left-handed, rotate the shorter wave-lengths much more than the longer wave-lengths, so that in turning the analyzer in the direction of rotation of the plane of vibration the longer wave-lengths are cut off first, leaving a blue tint, and shorter and shorter wave-lengths are cut off as the analyzer is rotated further and further. When medium wave-lengths are cut off the light which is transmitted is a mixture of red and violet which gives a purple tint, and when the shortest wave-lengths are cut off the transmitted light has an excess of red and yellow which gives an orange or yellow tint.

wise as seen by the observer) the angle β is equal to $O'S'$, or to $(O'S' + n \times 180°)$, where n is any whole number.   If, on the other hand, the sugar solution has turned the plane of polarization has turned the plane of polarization to the left, the angle β is equal to $O'S'$, or to $(O'S' + n \times 180°)$.

Knowing the direction of rotation from the test (3) above, the choice can be made between these two expressions for the value of β, and the value of n can be determined from the result of test (4).
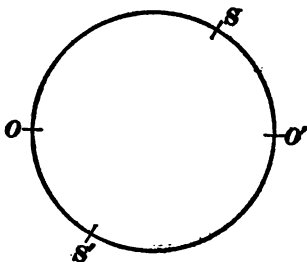
Fig. 192.

From the value of β so determined, calculate the sugar content s of the solution using equation (ii) above.

EXPERIMENT 130.

VELOCITY OF SOUND IN BRASS.

The object of this experiment is to determine the velocity of longitudinal waves along a brass rod in terms of the velocity of sound in air.

Apparatus and work to be done. — A brass rod R is clamped at its middle, and one of its ends projects into the open end of a glass tube as shown in Fig. 193.   In the other end of the tube
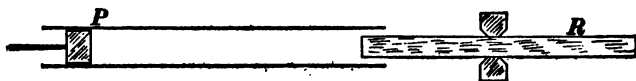
Fig. 193.

is a close-fitting piston P by means of which the length of air column may be adjusted.

Clean the tube, dry it by warming it slightly over a Bunsen burner, place a small quantity of lycopodium powder in the tube, and allow it to run the entire length of the tube.   Set up the apparatus as shown in Fig. 193, rub the end of the brass rod with a rosined cloth, thus setting it into violent longitudinal vibration,

and adjust the piston P until the air column vibrates in unison with the rod as indicated by the sweeping of the lycopodium powder into a series of equidistant heaps.

Determine the distance l between the lycopodium heaps by measuring over as many as can be easily distinguished, and measure the length L of the brass rod.

Computations and results. — The velocity of longitudinal waves of the rod is equal to L/l times the velocity of sound in air, and the velocity of sound in air is given by the equation

$$v = 331 \sqrt{1 + 0.0037t} \qquad \text{(i)}$$

in which v is the velocity of sound in meters per second, and t is the temperature of air on the centigrade scale.

Calculate the velocity of sound waves in brass.

EXPERIMENT 131.

Determination of Musical Pitch by Means of the Monochord.

The object of this experiment is to determine the pitch of a given musical tone by adjusting a vibrating string into unison with the given tone and calculating the pitch of the string from its length, tension, and mass per unit-length.

Theory. — The number of vibrations per second f of a stretched wire or string is given by the formula

$$f = \frac{n}{2l} \sqrt{\frac{T}{m}} \qquad \text{(i)}$$

in which n is the number of vibrating segments into which the wire divides, l is the length of the string in centimeters, T is the tension of the wire in dynes, and m is the mass in grams of one centimeter of the wire. The frequency f of the vibrations of the string can be calculated from this formula when n, l, T and m are known.

The monochord is an arrangement in which a wire is stretched

over a sounding board by a known tension by means of a weight hanging over an approximately frictionless pulley. The vibrating length of the string can be adjusted by means of a sliding bridge, so that the pitch of the musical tone produced by the string may be adjusted to unison with a given musical tone of which the pitch is to be determined.

Work to be done. — Apply a known tension T to the wire by means of weights of known mass, and adjust the length l of the wire by moving the sliding bridge until the wire vibrates in unison with the tone of which the pitch is to be determined. Then measure l.

Ask the instructor concerning the value of m.

Repeat these adjustments for several tuning forks and calculate the value of f for each.

EXPERIMENT 133.

DETERMINATION OF MUSICAL PITCH BY MEASURING WAVE-LENGTH.

The object of this experiment is to determine the pitch of a tuning fork by measuring the wave-length of the sound produced by it.

Method (a). — A vertical glass tube AA, Fig. 194, is connected to a vessel V so that by raising or lowering the vessel V the length of the air column in A can be varied at will. Place the rubber tube T in the ear, hold a vibrating tuning fork over the mouth of the tube AA, adjust the vessel V to a series of positions which give minimum loudness of sound, and observe the readings of the water level in the tube AA on the scale SS.

The distance between two adjacent readings on the scale SS as above determined is $\frac{1}{2}\lambda$, where λ is the wave-length of the sound produced by the tuning fork. The number of vibrations of the tuning fork per second may then be calculated by dividing the velocity of the sound by the value of λ so determined. See

Experiment 128 for an equation giving the velocity of sound in air.

The tube AA should be about 30 millimeters inside diameter, and three or four feet long ; and the pitch of the tuning fork should be between 300 and 400 vibrations per second.

The minimum of loudness occurs when the point of attachment of the rubber tube T is at the middle of a vibrating segment of the air column.   It is desirable to eliminate the sound reaching
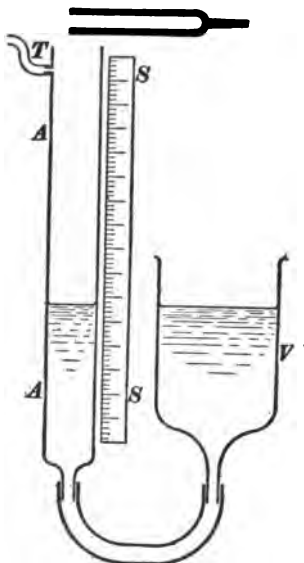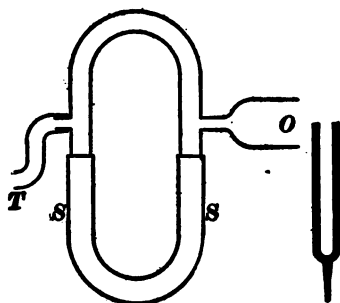
Fig. 194.   Fig. 195.

the ears directly from the tuning fork, and therefore the tube T should be placed in one ear, and the other ear should be closed with the finger or with a wad of cotton.

Method (b). — Hold a vibrating tuning fork in front of the opening O of a brass tube arranged as shown in Fig. 195, adjust the slider S to two adjacent positions for which the loudness of the sound is a minimum, and measure the distance d between these two positions of the slider.   The wave-length of the sound

produced by the tuning fork is equal to $2d$ and the number of vibrations per second of the tuning fork may be calculated as explained above.

Work to be done. — Determine the pitch of a tuning fork by each of the above methods, making ten or more settings for each and determine the probable error of each result.

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